THE UAV CONTINUOUS COVERAGE PROBLEM

THESIS

Taegyun Ha, Captain, Republic of Korea Army

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DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Taegyun Ha, BS

Captain, Republic of Korea Army

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Taegyun Ha, BS
Captain, Republic of Korea Army

Approved:

__________//SIGNED//__________________	23 MARCH 2010_
Dr. J. Kebir	date
__________//SIGNED//__________________	23 MARCH 2010_
Dr. J. Cochran	date

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Abstract

The purpose of this research is to develop a method to find an optimal UAV cyclic schedule to provide maximum coverage over a target area to support an ISR mission. The goal is to reach continuous coverage. UAV continuous coverage of a target area is crucial for the success of an ISR mission. Even the smallest coverage gap may jeopardize the success of the mission. Ideally it is desirable to obtain continuous coverage of a target area but the stochastic nature of the problem makes continuous coverage without gaps unlikely. However, it is still possible to obtain a high coverage rate. Coverage gaps may occur at handoff from one UAV to another. We first study a deterministic model with identical UAVs and obtain the minimum number of required UAVs to ensure continuous coverage. Continuous coverage is possible only in the deterministic setting. The model provides valuable insights on the parameters driving the UAV performance coverage. We show that the loitering and the roundtrip times are the most impacting parameters driving the performance coverage of the UAVs. We prove that the number of UAVs is an increasing function of the roundtrip time and a decreasing function of the loitering time. We show that a minimum cyclic scheduling emerges in a natural way when the fleet consists of identical UAVs. The results obtained for the model with identical UAVs are then extended to the deterministic model with possibly non-identical UAVs. Again because it is a deterministic model, continuous coverage can be achieved. Conditions for continuous coverage are obtained and used to formulate the scheduling problem as an integer linear programming model. Special cases of the deterministic model are also

studied and conditions ensuring continuous coverage are given. Similar results for the stochastic model are obtained. The stochastic model can be formulated as a stochastic programming model with probabilistic constraints. Also, special cases are studied where the UAV attributes have specific probability distributions. Results obtained can be applied to other surveillance problems and particularly those pertinent to NRO and NSA.

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To my wife…

Acknowledgments

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Taegyun Ha

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Chapter 1

I. INTRODUCTION

1.1 Setting

In the past two decades, conventional warfare has shifted towards to asymmetric warfare as witnessed in the Iraqi war. Conventional warfare often proves to be ineffective in such situations and new effective strategies to fight vigorously the enemy in these new environments and realities is a necessity. In particular, developing and acquiring an asymmetric warfare capability is regarded as an important tool for the armed forces to fight effectively. In the asymmetric warfare framework, adversaries tend to use basic and rudimentary but nevertheless effective methods that conventional warfare technology does not know how to counter. It is inadequate and ineffective to fight with conventional weapons an enemy that uses tactics such as camouflage, improvised explosive devices, suicide bombings, hiding among the civilian population, hiding in rugged terrains hard to access and so on. As a result, there is a definite need for some innovative means to help the war-fighter fight asymmetrically and, not surprisingly, it is here that the Unmanned Aerial Vehicle (UAV) comes in handy.

The Unmanned Aerial Vehicle idea, motivation, and development have a long history but, for our purpose, it suffices to say that its accelerated development and deployment were motivated by the recent need to be an effective player in the asymmetric warfare arena.

The USAF and the US Department of Defense, over the past two decades, have been using UAVs successfully to fight back by aggressively using them to track and attack the enemy. It turned out, as attested by various reports pertinent to the Iraq, Bosnia, Kosovo and Afghanistan wars that the UAV has proved itself to be an effective and lethal tool to combat the enemy. It is no surprise that the US Department of Defense made the UAV an integral part of the military capabilities and has dedicated substantial funds to support its development and integration into National Defense (1:37, 2:26). Various branches of the military use the UAV as an ISR tool in seeking supremacy over the adversary. Homeland Security also uses UAVs as a valuable tool to control illegal immigration and fight drug trafficking.

Using UAVs to continuously watch the enemy and acting at the right time pays off. Recently, in Iraq, the UAV was used to track the leader of Al-Queda and ultimately eliminate him. The effective tracking was the result of carefully watching his whereabouts for 600 hours continuously (3). The <u>*continuous*</u> surveillance and coverage by the UAVs was the key for the success of the operation. Indeed, were it not for the UAV continuous coverage of the target, the enemy could have easily escaped during a UAV coverage gap (i.e., surveillance interruption) thus rendering the whole tracking operation a failure. It is therefore easy to understand why UAV continuous coverage is so critical for the success of crucial ISR missions.

This research is about UAV continuous coverage and surveillance optimization. More specifically, it is about optimally scheduling UAVs cyclically to carry out a critical ISR mission that requires continuous coverage of the target area. The motivation is to provide

surveillance without interruption because the purpose of the mission depends on it. The ultimate goal is to provide a continuous flow of information and data to support the war-fighter on the ground, to enhance the global efforts on fighting terrorism, and to improve the asymmetric warfare capabilities of the armed forces.

1.2 Problem Statement

A critical ISR mission requiring continuous surveillance and coverage of a target area is to be accomplished using UAVs as the main resource. A UAV fleet is available at the operating base to support the mission. Because the UAV is a valuable and scare resource it has to be used parsimoniously particularly when there are other ISR missions to be conducted at other sites around the world as it is often the case. The manager responsible for the mission needs to assemble a team of UAVs to carry out the mission. The main questions that need to be answered are how to sequence the UAVs to conduct the mission to provide continuous coverage of the target area and how many UAVs are needed knowing that one should not use more UAVs than needed. In short the manager is faced with a scheduling optimization problem where he seeks to find the best UAV cyclic schedule to provide coverage without interruption of the mission because the success of the mission depends on it. The manager needs help to answer his questions in a general setting and is particularly interested in developing a mathematical model to derive structural results and insights to guide his decision making process and it is not so much interested in simulation at this point. The manager stresses that continuous coverage is a key requirement. Ideally, if possible, he would like to see no coverage gaps at all since

that may render the mission worthless; for example, objects of interest may move out of the target area without him being aware of it. The manager realizes that there could be several unforeseen events that could prevent continuous coverage and if it is the case then he wants to obtain the maximum coverage possible.

The manager's problem which is simply stated as the *UAV continuous coverage problem* is the main focus of this thesis. More specifically, the purpose of this thesis is to develop a UAV continuous coverage scheduling optimization model to help the manager figure out how to come up with an optimal UAV cyclic schedule that provides continuous coverage of the target.

1.3 Objective

The goal of this research is to establish a mathematical modeling foundation for the UAV scheduling and coverage problems. A mathematical baseline structure is needed so that further research will build on it to tackle more complex UAV scheduling and coverage problems. This research is a first step toward achieving that. It does not lean much on previous work since the basic ideas and approach are new. The UAV continuous coverage problem is indeed complex. Here we simplify the problem so that the understanding and insights we gain from the basic version of the problem can be extended to more complex versions. The main tasks to be competed are:

1. Develop an adequate mathematical framework for the UAV continuous coverage problem by starting with a very basic version of the problem.

2. Develop a deterministic model for a homogenous fleet of UAVs and derive necessary and sufficient conditions for the optimality of cyclic schedule.
3. Develop a deterministic model for non-homogenous fleet of UAVs and derive a method that finds an optimal cyclic schedule.
4. Derive a stochastic programming approach to find an optimal cyclic scheduling when some of the UAV basic data is stochastic.

1.4 Scope

This research is the first of its kind. To the best of our knowledge, the mathematical model developed in this thesis is introduced here for the first time. The continuous coverage model developed in this work can be applied to other situations where a task is to be processed continuously without interruptions and the "agents" providing the resources to perform the task are scheduled cyclically. Each agent carries out a portion of the task before handing it over to the next one. Here the agent is limited in its capability to work for a long time without interruption because it needs resources to sustain itself while working and so needs to break away from the task while another agent takes over. Therefore, an important characteristic of the task to be executed (processed) is that only one agent can work on the task at a time. In other words, because of the nature of the task at hand, the agents are not allowed to work concurrently. Search and rescue missions where continuous coverage may be crucial to find survivors, aerial tankers needing to orbit while waiting to refuel aircraft, satellite orbiting to provide a continuous flow of information may be modeled using the results of this thesis. We anticipate that the

present work may serve as a baseline for further work in UAV scheduling and continuous coverage.

1.5 Overview

Chapter 2 presents the mathematical setting and the basic definitions needed for modeling the UAV continuous coverage problem and then proceeds to study a simple version of the problem. The first model referred to as the *basic model* studies the UAV continuous coverage problem where the operational UAV fleet is homogenous in the sense that all the UAVs are identical. The main ideas, insights, and approach from this basic model will be the foundation for studying more complex versions of the basic model. In fact, this approach of first studying a simple version of a scheduling problem and then extending the insights to more complex problems has been used quite often in scheduling theory. The main results we derive for the basic model are necessary and sufficient conditions that characterize the optimal cyclic schedule. Also, we derive monotonicity properties to show how the optimal number of UAVs depends on the UAV attributes. Next, in Chapter 3, we extend the results of Chapter 2 to the case of a non-homogenous fleet of operational UAVs where the UAVs are not necessarily identical with respect to their attributes. Here the model is a more complex combinatorial optimization problem which we formulate as a binary integer linear programming. This problem is computationally NP hard and therefore does not have an efficient algorithmic solution when the size of the UAV fleet is very large (4:13, 34). However, for small size problems traditional techniques such as branch and bound methods can be used for numerical

applications. With the intent of solving large scale UAV coverage models we introduce the notion of a minimal cyclic schedule. The concept of a minimal cyclic schedule can be used in a Tabu search procedure for example because non-minimal cyclic schedules are not optimal and therefore can be excluded from the search. Since the problem is NP hard, heuristics can be used to find good solutions for large scale models. Finally the model is extended to the case where some UAV attributes are stochastic. Here the problem is formulated as a stochastic program because an imperfect handoff may occur with a positive probability. The models have also been extended to account for an admissible coverage gap of some length deemed not to affect the mission objective.

Chapter 2

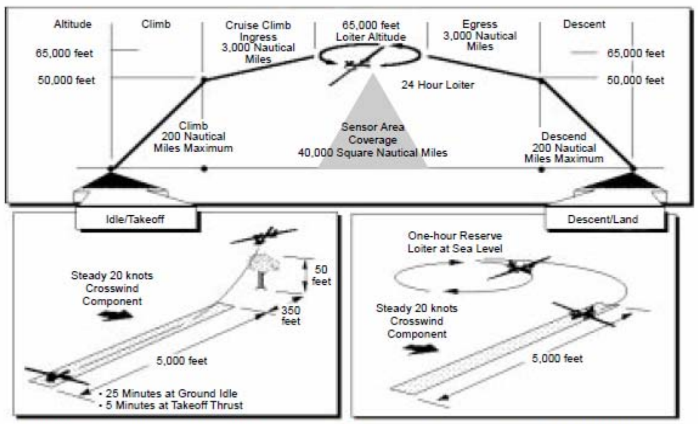
II. THE MODEL FOR A HOMOGENEOUS UAV FLEET

2.1 Introduction

The UAV continuous coverage model introduced in this thesis is new and does not lean much on previous research because no modeling work has been done on UAV continuous coverage in the same spirit it is done here. There is a vast amount of research done on UAVs but very little is directly related to the present work. Related UAV studies have appeared in the area of UAV decision and control (5), UAV swarms (6, 7, 8), and UAV simulation (9) but these studies do not have any direct impact on the continuous coverage problem approach as it is conceived here. However, for a general background the references that the reader will find very useful are Fahlstrom and Gleason (10), Howard (11), Renehan (12), Longino (13), Kennedy (14) and Stephenson (15). Other US Government documents and reports which are helpful for a basic understanding of UAVs are (1, 2, 16, 17).

A UAV system consists of several components such as the ground control station, launch and recovery, payload, data links etc. However, we simply focus on the air vehicle part which we refer to as the UAV. Hence for the purpose of the study the UAV is just a flying object. One very important aspect of the UAV though is its flight trajectory which plays a major role in the search for the best cyclic schedule. In an ISR mission a UAV will be commuting between the operating base and the target area to support the mission. Moving the UAV back and forth between the operating base and the target area may be

regarded as a supporting activity whereas loitering is the core task (activity). Generally then the performance of the UAV will be based on how much support is provided to the core task. For the same loitering time the less support provided the better. An idea of productivity based of the ratio of support to core expenses will be reflected in the result that we derive.



Source: Extracted from the Air Combat Command, *Endurance Unmanned Aerial Vehicles Concept of Operations*, 3 December 1996–version 2.

Figure 1. The UAV Flight Trajectory

Although the trajectory of the UAV during an ISR mission can be complex (see Figure 1) we simplify it to a few phases. The UAV starts from a mission ready state at the operating base, flies to the target area, loiters for a pre-specified length of time, then hands over the mission to the next scheduled UAV and heads back to the operating base. The UAV periodically goes back to the target area to provide coverage. Because the UAV has a limited endurance time, it cannot loiter indefinitely over the target area and thus needs to break away from the orbiting task. A UAV ending its loitering tour returns to the operating base for refueling, inspection, and maintenance. In fact, refueling is the major reason the UAV returns to the operating base. We assume all along that the mission length is longer than a UAV endurance time so that more than one UAV are

needed to provide continuous coverage since obviously one UAV alone cannot provide continuous coverage. Therefore, several UAVs need to work collaboratively as a team to successfully accomplish a critical ISR mission requiring continuous coverage by relaying each other cyclically.

The goal is to build the best UAV cyclic schedule to provide continuous coverage of a target area. A UAV cyclic schedule consists of a finite number of UAVs scheduled sequentially (serially) to provide continuous coverage. In a cyclic schedule, each UAV provides coverage periodically. The objective is to provide coverage without interruptions with a minimum number of UAVs or equivalently to find the optimal cyclic schedule that provides coverage with no gaps of the target area. The concept of cyclic scheduling (18, 19, 20, 21, 22) has been used in a production environment but the UAV continuous coverage problem cannot be cast within those standard frameworks of scheduling theory because the present objective may not have a meaningful interpretation in a production environment.

In this chapter we study a deterministic UAV continuous coverage model with identical UAVs. The model is referred to as the basic model.

2.2 Preliminaries and Basic Definitions

The following definitions will be used throughout this thesis. A few of them will be made mathematically more precise as the work progresses. Formally, a <u>*target area*</u> (area of responsibility) is the area where the UAV conducts a surveillance mission. A UAV <u>*roundtrip*</u> is the time it takes the UAV to fly from the operating base to the target area and

back. This includes the time spent in maintenance, refueling and inspection**.** The *endurance* of a UAV is the maximum duration the UAV can sustain itself flying. It depends on several factors such the amount of fuel it carries, the weight of the payload, weather conditions, enemy hostility, etc. A *coverage gap (interruption)* occurs when the target area is left unwatched for some time however short that may be. Levels of coverage gaps and their severities may also be defined when the analysis calls for it. The coverage of a target area is said to be *continuous* or *uninterrupted* if no coverage gap (interruption) occurs during the mission. A UAV is said to be *operationally (mission) ready* at time t if at time t it can take over the surveillance mission from another UAV and start loitering over the target area at time t. A *perfect handoff* occurs when a UAV hands over the mission to the next UAV with no coverage gap. An *imperfect handoff* occurs when there is a coverage gap at the time of handoff. We say that a UAV *takes over the mission successfully* from the active and departing UAV if no coverage gap is induced at handoff. The *loitering (loiter) time* is the time the UAV spends orbiting over the target area collecting intelligence data. A UAV schedule is *feasible* if it does not cause coverage gaps during the mission. The *slack time* of a UAV is the time between the time UAV is mission ready to the time it is deployed to the target area.

To each UAV, we assign two *attributes*. The first one is the roundtrip time and the second one the loitering time. Let the fleet with K UAVs be represented by a finite set $F = \{(\Delta_1, T_1),\ (\Delta_2, T_2),\ \dots, (\Delta_K, T_K)\} = \{V_1, V_2, \dots, V_K\}$ where $V_i = (\Delta_i, T_i)$ is the *attribute vector* of UAV$_i$. The time Δ_i is the roundtrip time and T_i the loitering time of UAV$_i$. The roundtrip time Δ_i is the aggregation of three components. It is the sum of

three variables ξ_1^i, ξ_2^i and ξ_3^i where ξ_1^i is the trip duration from the operating base to the target area, ξ_2^i the trip duration from the target area to the operating base, and ξ_3^i the duration spent in maintenance and repair at the operating base.

Definition 2.2.1 *Let the precedence symbol "$\rightarrow$" be the "mission handoff" symbol where we write* $V_i \rightarrow V_j$ *to mean that UAV$_i$ hands over the mission to UAV$_j$.*

Definition 2.2.2 *Let the precedence symbol "$\xrightarrow{s}$" be the "successful mission handoff" symbol where we write* $V_i \xrightarrow{s} V_j$ *to mean that UAV$_i$ hands over the mission to UAV$_j$ successfully.*

Definition 2.2.3 *The ordered sequence of UAVs* $E = \left(V_{i_1}, V_{i_2}, \dots, V_{i_k}\right)$ *with* $V_{i_j} = \left(\Delta_{i_j}, T_{i_j}\right)$, $j = 1,2,\dots,k$ *is said to cyclic schedule if :*

(a) $V_{i_1} \rightarrow V_{i_2} \rightarrow \cdots \rightarrow V_{i_{k-1}} \rightarrow V_{i_k}$

(b) $V_{i_k} \rightarrow V_{i_1}$

A schedule is said to be of size 6 if it uses 6 UAVs. Figure 2 shows a graphical representation of a cyclic schedule with 6 UAVs as a circuit with 6 nodes with one for each UAV.

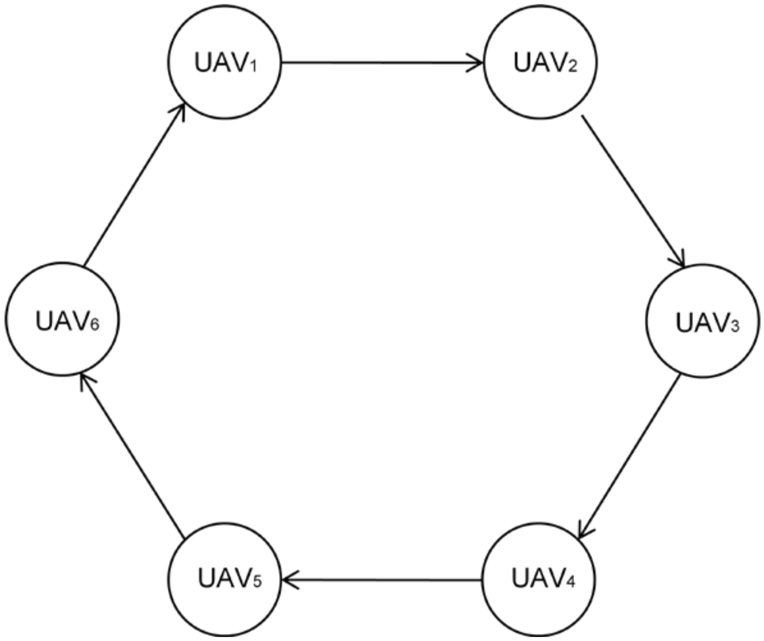


Figure 2. Cyclic Schedule with 6 UAVs

Each oriented arc symbolizes a mission handoff from one UAV to the next. The way this schedule of size 6 works is straightforward. First UAV_1 flies to the target area to loiter for a duration equal to T_1 and heads back to the operating base. UAV_2, which is supposed to be at the target area when UAV_1 is about to be done, takes over the mission and begin its loitering task which lasts T_2. Then the same happen between UAV_2 and UAV_3 etc. When UAV_6 finishes its share of loitering it hands over the mission to the very first UAV_1 and a scheduling cycle has been accomplished. The next cycle is similar to the previous one. Note that a UAV is used only once during a scheduling cycle and there is only one loitering UAV over the target area. A cyclic schedule gives each UAV a certain “rest” time away from the target area. This rest period consists mainly in refueling and undergoing the necessary maintenance and inspection to get the UAV in “top shape” again for its next loitering tour.

2.3 The Basic Model

The UAV model with a homogeneous fleet of identical UAVs is referred to as the ***basic model***. In this basic model, the roundtrip and loitering times of the UAVs are all equal to (Δ, T) and the UAV fleet can be written as $F = \{(\Delta, T), (\Delta, T), \ldots, (\Delta, T)\}$ with $|F| = K$. Note that we are dropping the subscript when writing the attributes of a UAV. Define the following variables h, d, T, and Δ as being respectively the loitering altitude over the target area, the distance between the operating base and the target area, the loitering time, and the roundtrip time of the UAV. Recall that $\Delta = \xi_1 + \xi_2 + \xi_3$.

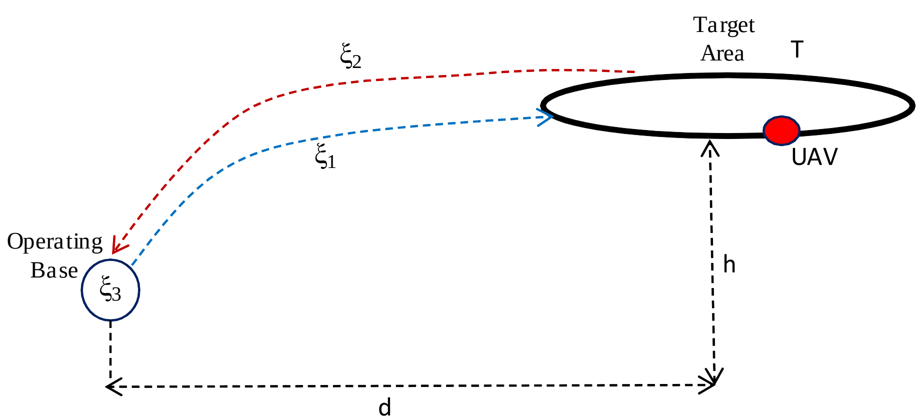


Figure 3. UAV Flight Trajectory Components

Figure 3 shows the main elements of the basic model. It shows the flight trajectory of the UAV flying up to the target area and back. The roundtrip and loitering times (Δ and T respectively) of a UAV clearly depend on the distance d and loitering altitude h.

To illustrate the main idea of when a cyclic schedules may generate coverage gaps we consider two specific cyclic schedules where the first one does not generate coverage gaps whereas the second one does. Basically is is about comparing the roundtrip time of a UAV to the aggregated loitering times of the other UAVs of the cyclic schedule.

Let us start with a cyclic schedule that does not induce coverage gaps. As mentioned earlier such a schedule should consists of at least two UAVs. More specifically, consider a cyclic schedule consisting of two identical UAVs with the following parameters:

$T = 8$ hours,

$\xi_1 = 2$ hours,

$\xi_2 = 2$ hours, and

$\xi_3 = 4$ hours.

Figure 4 shows these two UAVs working together in such a perfect unison that they do not cause coverage gaps.

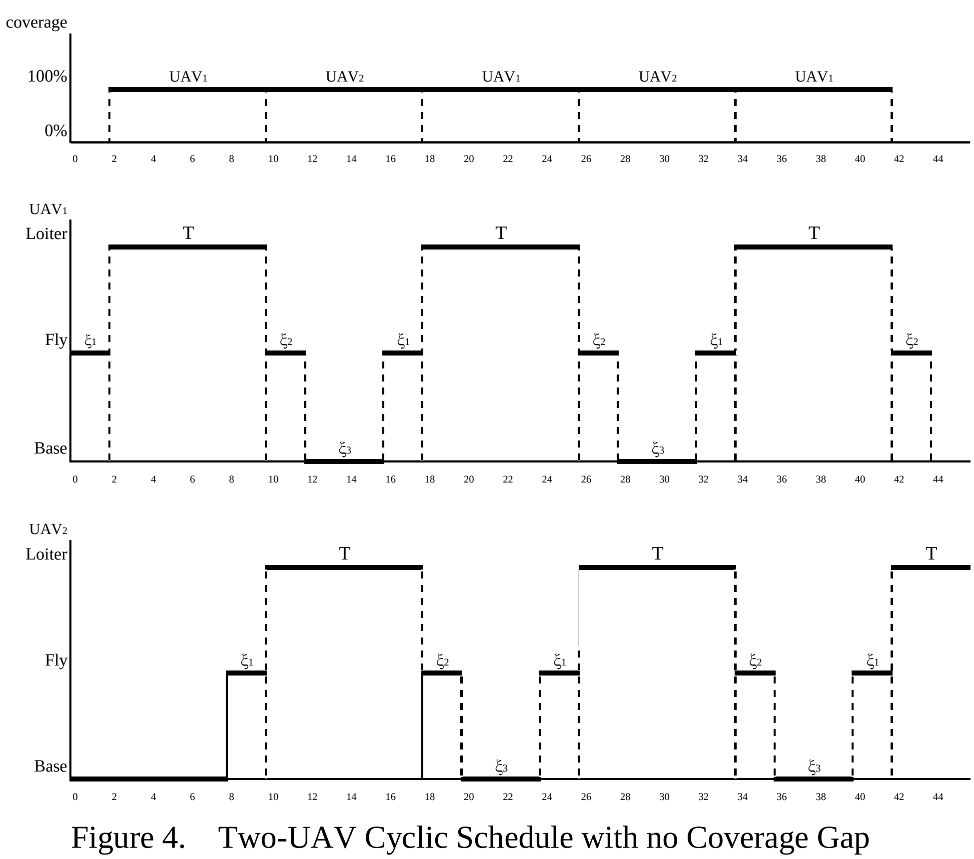


Figure 4. Two-UAV Cyclic Schedule with no Coverage Gap

The reason that this size 2 cyclic schedule is free of coverage gaps is as follows. At time 2 UAV_1 starts its loitering tour over the target area and at time 10 finishes its loitering tour and needs to leave the target area. In the first cycle, it is clear that UAV_2 can take over the mission successfully. At time 10 a perfect handoff occurs smoothly from UAV_1 to UAV_2. We assume that the handoff process is perfect and takes a negligible length of time. That is, as UAV_1 moves out of the target area UAV_2 moves in. Next, at time 18 UAV_2 finishes its loitering task and needs to leave the target area. At that time UAV_1 can take over the mission successfully from the UAV_2 because the roundtrip time of UAV_1 is the same as the loitering time of UAV_2 which means that while UAV_2 loiters over the target area UAV_1 is capable of flying to the operating base and back to the target area. As a result, a perfect handoff from UAV_2 to UAV_1 takes place. Clearly in order to avoid a coverage gap the roundtrip time of UAV_1 needs to be smaller than the loitering time of UAV_2 and similarly the roundtrip time of UAV_2 needs to be smaller than the loitering time of UAV_1. But UAV_1 and UAV_2 being identical have the same loitering and roundtrip times and therefore to avoid coverage gaps we must have:

$$\xi_1 + \xi_2 + \xi_3 \leq T.$$

Let us consider another cyclic schedule scenario with a farther target area having the following parameters:

$T = 8$ hours,

$\xi_1 = 6$ hours,

$\xi_2 = 6$ hours, and

$\xi_3 = 2$ hours.

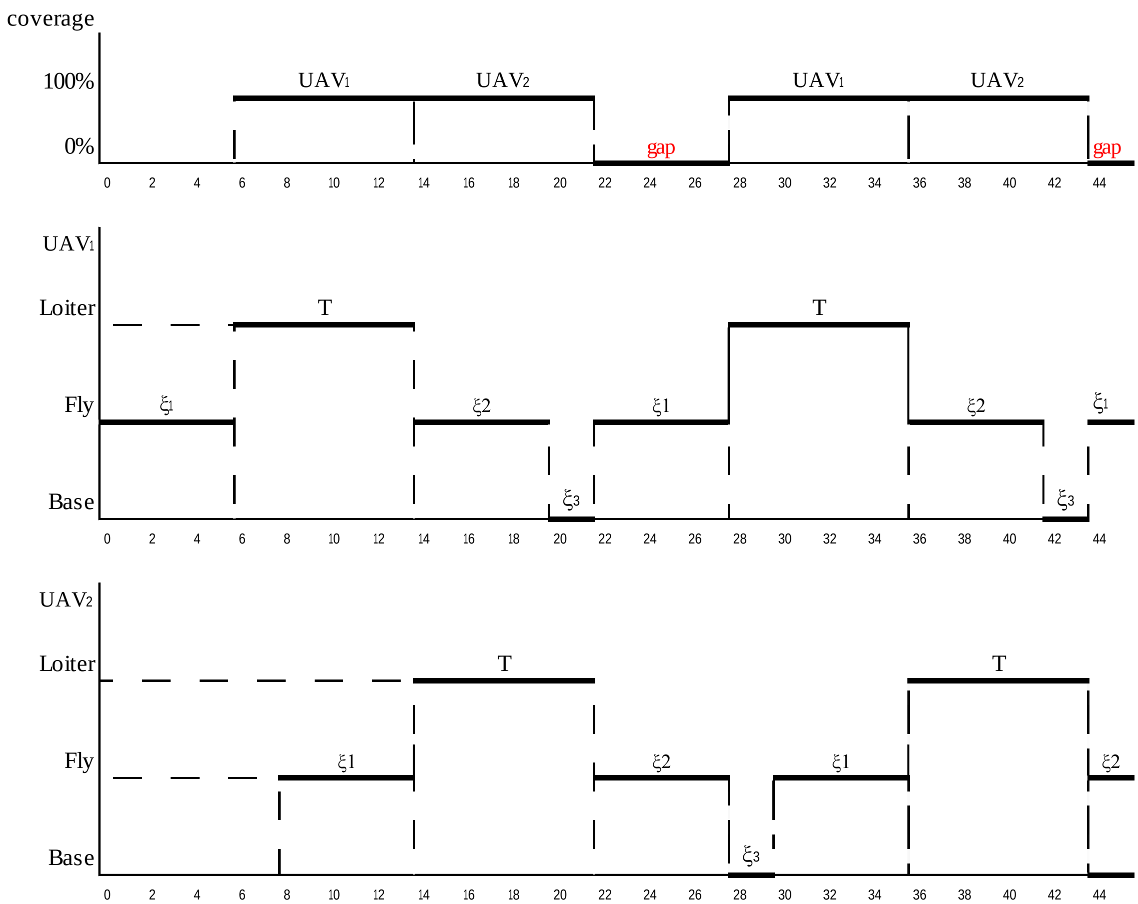


Figure 5. Two-UAV Cyclic Schedule with a Coverage Gap

Figure 5 shows how the schedule dynamically unfolds. It is clear that $\xi_1 + \xi_2 + \xi_3 > T$. As a result, a coverage gap is generated by this cyclic schedule. Figure 5 can help understand why such a schedule causes a coverage gap. By the time UAV_2 finishes its loitering tour UAV_1 has not been able to reach to target area because of the length of its roundtrip time Δ and, as a result, a coverage gap occurs during the switch from UAV_2 to UAV_1. Thus, to avoid coverage gaps one additional UAV at least is needed. Let us add one more UAV to have a schedule with three identical UAVs and the same performance parameters as earlier. This time in order to show how this new size 3 cyclic schedule

unfolds we use a Gantt chart because it is more effective and easy to read. Figure 6 depicts the cyclic schedule with 3 UAVs as a Gantt chart.

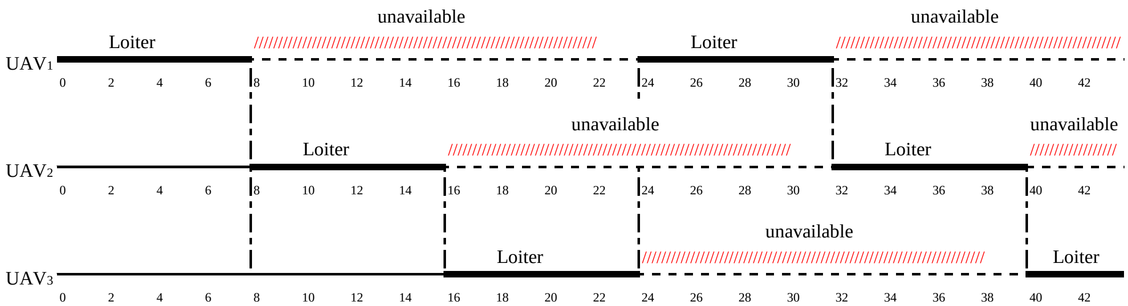


Figure 6. Three-UAVs Cyclic Schedule with no Coverage Gap

By time 24 all three UAVs have loitered once. At time 24 when UAV_3 is done with its loitering task we check whether one of UAV_1 or UAV_2 is able to take over the mission without causing a coverage gap. Clearly UAV_2 cannot assure that since time 24 falls into its shaded area which represents its unavailability time (roundtrip time) period; however, UAV_1 is available at time 24. Consequently, in order for UAV_1 to be able to take over the mission successfully from UAV_3 it must that its roundtrip time should not last longer than the aggregated loitering times of UAV_2 and UAV_3. This means that UAV_1 should be able to return to the target area before UAV_2 and UAV_3 are done with their loitering tasks. Thus to obtain continuous coverage we must have:

$$\xi_1 + \xi_2 + \xi_3 \leq 2T.$$

Since the three UAVs are identical this inequality holds for all of them. Next, consider a new target area which is farther from the operating base than the previous one and with the following parameters:

$T = 7$ hours,

$\xi_1 = 7$ hours,

$\xi_2 = 7$ hours, and

$\xi_3 = 2$ hours.

Here, the previous inequality which led to continuous coverage does not hold anymore because $2 \times T = 14$ is smaller than $\xi_1 + \xi_2 + \xi_3 = 16$ and so this schedule will cause coverage gaps. Therefore, at least one additional UAV is needed for this schedule to have the potential to provide continuous coverage.

We have analyzed a few specific cyclic scheduling scenarios to find out the reasons a schedule may or may not provide continuous coverage. An idea that plays a major role is whether or not a departing UAV from the target area can be back to the area before the other UAVs of the cyclic schedule have each done their share of loitering. The results of this section are based on that idea.

2.4 Results

The first main result for the model with identical UAVs is a necessary and sufficient condition that characterizes the optimal cyclic schedule. The result provides a formula for the minimum number of identical UAVs required to support a mission with continuous coverage. Two more monotonicity properties show how coverage depends on the roundtrip and loitering times. More specifically, if T is kept constant and $\xi_1 + \xi_2 + \xi_3$ increases (decreases) then the minimum number of UAVs needed for continuous

coverage increases (decreases). Also, as the loitering time T increases (decreases) the minimum number of UAVs decreases (increases).

2.4.1 Main Result

Theorem 2.4.1 *Let the positive integer* $N \geq 2$ *be such that*

$$(N-2)T < \Delta \leq (N-1)T.$$

Then, a schedule with N *UAVs ensures a continuous coverage. However, a schedule with* $N-1$ *or less UAVs causes a coverage gap during the mission.*

Proof. We need at least 2 UAVs to obtain continuous coverage, and thus $N \geq 2$. We proceed to prove the assertion of the theorem by induction. First, we show the result for $N = 2$. Assume that

$$0 < \Delta \leq T \tag{2.4.1}$$

where, we recall that $\Delta = \xi_1 + \xi_2 + \xi_3$. We show that with exactly $N = 2$ UAVs the mission can be accomplished with no coverage gap. The following chart shows a cyclic schedule with 2 identical UAVs.

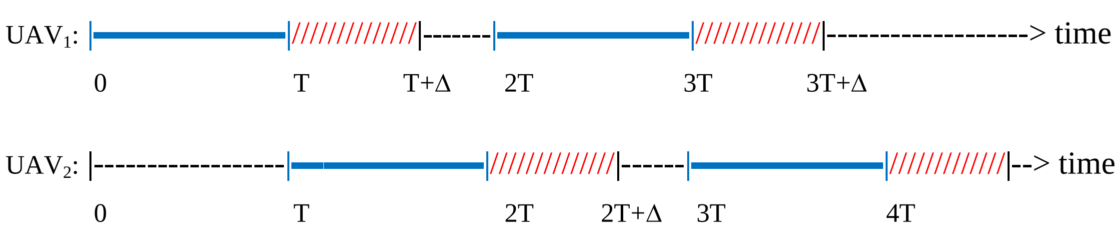


Figure 7. Cyclic Schedule with 2 UAVs

Note that the scheduling cycles are $[0,2T]$, $[2T,4T]$, $\cdots$. UAV_1 loiters from time zero until time T whereas UAV_2 does it from time T to $2T$. At time $2T$ when UAV_2 is

done with its ISR tour, UAV_1 can take over the surveillance with no coverage gap if and only if UAV_1 is available at time $2T$. But UAV_1 is available the earliest at time $T+\Delta$. Thus UAV_2 can take over the mission successfully if and only if

$$T+\Delta \leq 2T$$

or, equivalently

$$0<\Delta \leq T$$

which is exactly condition $(2.4.1)$. Next, let UAV_1 take over the mission at time $2T$ and loiter from time $2T$ to $3T$. Now we check whether UAV_2 can take over the mission successfully from UAV_1. The answer is affirmative because the inequality $2T+\Delta \leq 3T$ is equivalent to $\Delta \leq T$. UAV_2 will loiter in $[3T, 4T]$ and at time $4T$ UAV_1 takes over. Then a new identical cycle starts all over again. Therefore, the theorem is proved when $N=2$. Next, we show that the theorem is true when $N=3$. Assume that $T< \xi_1 + \xi_2 + \xi_3 \leq 2T$ or

$$T< \Delta \leq 2T. \tag{2.4.2}$$

First, $T< \Delta$ implies that condition $(2.4.1)$ is violated and consequently we need more than two UAVs to ensure continuous coverage. We show that we need exactly 3 UAVs. With 3 UAVs we have the following (Gantt) chart:

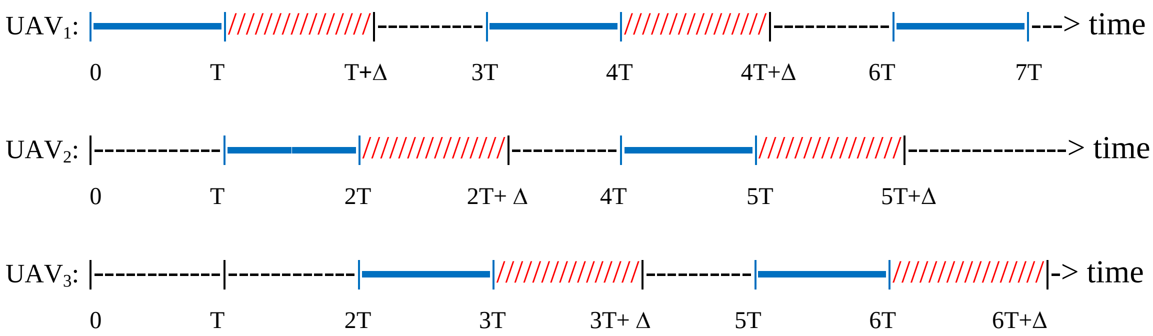


Figure 8. Cyclic Schedule with 3 UAVs

Note that the scheduling cycles are $[0, 3T]$, $[3T, 6T]$, …, and we have the following sequence of events:

UAV_1: Loiters from time 0 to time T and is operationally ready the earliest at time $T + \Delta$.

UAV_2: Loiters from time T to time $2T$ and is operationally ready the earliest at time $2T + \Delta$.

UAV_3: Loiters from time $2T$ to time $3T$ and is operationally ready the earliest at time $3T + \Delta$.

⋮

When UAV_3 completes its surveillance tour at time $3T$, either UAV_2 or UAV_1 takes over. Notice that UAV_2 cannot take over without causing a coverage gap because it is not operationally ready at that time. In fact, UAV_2 is operationally available the earliest at time $2T + \Delta$ and thus UAV_2 can take over from UAV_3 with no coverage gap if and only if

$$2T + \Delta \leq 3T$$

or

$$\Delta \leq T$$

However, we know that $\Delta > T$ from (2.4.2). As a result, UAV_2 cannot ensure continuous coverage. It remains to check that UAV_1 can take over the mission from UAV_3 without a coverage interruption. The question is whether UAV_1 is available at time $3\mathrm{T}$ when UAV_3 is done with its tour. In fact since UAV_1 is available at time $\mathrm{T} + \Delta$, it can take over successfully if and only if

$$T + \Delta \leq 3T$$

or

$$\Delta \leq 2T$$

But, this last inequality is part of (2.4.2) and thus UAV_1 can take over successfully. Now assume that UAV_1 takes over the mission and let it loiter in the time interval $[3T, 4T]$. Then, at time $4T$ when UAV_1 is done with its tour we check which of UAV_2 and UAV_3 can take over successfully. In fact UAV_2 can take over since $2T + \Delta \leq 4T$ is equivalent to $\Delta \leq 2T$. However, UAV_3 cannot take over the surveillance from UAV_1 because $3T + \Delta \leq 4T$ being equivalent to $\Delta \leq T$ contradicts $\Delta > T$. Now let UAV_2 take over and loiter in $[4T, 5T]$. A similar argument shows that only UAV_3 can take over the mission successfully from UAV_2 at time $5T$. Thus, let UAV_3 loiter in $[5T, 6T]$. Similarly, at time $6T$, only UAV_1 can take over the mission from UAV_3 with no coverage gap. Then another identical cycle starts all over again at time $6T$ and thus the theorem is proved for $N = 3$. In fact, for $N = 3$ UAVs we have shown that we have a feasible UAV schedule (see Figure 9) with three UAVs.

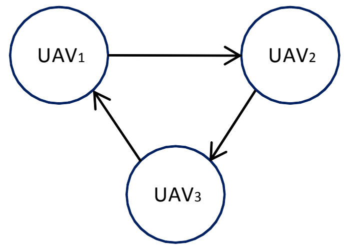


Figure 9. Cyclic Schedule of Size 3

Next, we show the theorem is true for any $N \geq 2$. For that, we assume the theorem is true up to $N - 1$ and prove that it is true for N. Assume that

$$(N-2)T < \Delta \leq (N-1)T. \tag{2.4.3}$$

Since $(N-2)T < \Delta$, then Δ does not belong to any of the intervals

$$[0, T], [T, 2T], \dots, [(N-3)T, (N-2)T]$$

and thus by the induction assumption the number of required UAVs must be larger than or equal to N to ensure continuous coverage. We show that exactly N UAVs are required. Note that the scheduling cycles are $[0, N\mathrm{T}]$, $[N\mathrm{T}, 2N\mathrm{T}]$, … and the following events take place:

UAV_1: Loiters from time 0 to time T and is operationally ready the earliest at time $T + \Delta$.

UAV_2 : Loiters from time T to time $2T$ and is operationally ready the earliest at time $2T + \Delta$.

UAV_3 : Loiters from time $2T$ to time $3T$ and is operationally ready the earliest at time $3T + \Delta$.

⋮

UAV_{N-1} : Loiters from $(N-2)T$ to time $(N-1)T$ and is operationally ready the earliest at time $(N-1)T + \Delta$.

UAV_N : Loiters from $(N-1)T$ to time NT and is operationally ready the earliest at time $NT + \Delta$.

These are shown in Figure 10.

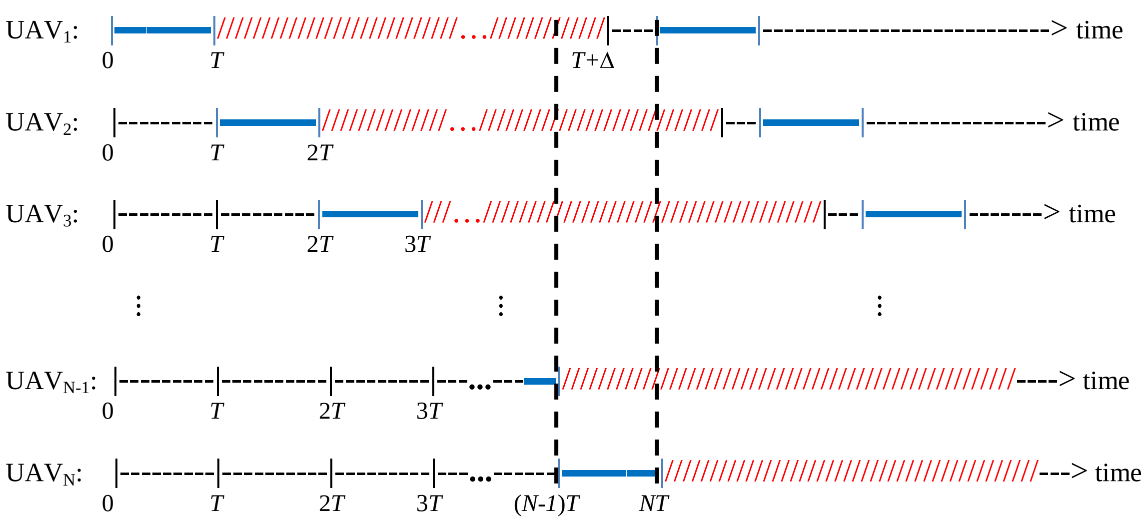


Figure 10. Cyclic Schedule with N Identical UAVs

UAV_N completes its first loitering at time NT. We show that at least one UAV from the collection {UAV_1, UAV_2, … , UAV_{N-1}} can take over the mission without causing a coverage gap. In fact, we show that UAV_1 is the only UAV available to take over the surveillance from UAV_N without causing a coverage interruption. First, UAV_2 is operationally ready the earliest at time $2T + \Delta$ and can take over with no coverage gap if and only if $2T + \Delta \leq NT$, or

$$\Delta \leq (N-2)T$$

But we know that $\Delta > (N-2)T$ and, as a result, UAV_2 cannot successfully take over the mission from UAV_N. Next, we show that UAV_3 cannot either. Indeed UAV_3 is available the earliest at time $3T + \Delta$ and therefore can take over with no coverage gap if and only if $3T + \Delta \leq NT$, or

$$\Delta \leq (N-3)T.$$

But we know that

$$\Delta > (N-2)T > (N-3)T$$

and so UAV_3 cannot take over successfully. We next prove that in general for $k, 2 \leq k \leq N-1$, UAV_k cannot take over from UAV_N without inducing a coverage gap. Indeed, UAV_k is able to take over the mission successfully from UAV_N if and only if $kT + \Delta \leq NT$ or

$$\Delta \leq (N-k)T, \;\; 2 \leq k \leq N-1.$$

But we have

$$\Delta > (N-2)T > (N-3)T > \cdots > (N-k)T,$$

therefore, when $2 \leq k \leq N-1$, UAV_k cannot take over the mission without incurring a coverage interruption. It remains to show that UAV_1 can take over the mission with no coverage gap. In fact UAV_1 is the only one that can do so. Indeed UAV_1 is available the earliest at time $T + \Delta$ and can take over the mission if and only if $T + \Delta \leq NT$, or $\Delta \leq (N-1)T$. But, this last inequality is given in (2.4.3), and so UAV_1 inherits the mission from UAV_N with a successful handoff. Let UAV_1 takes over successfully at time NT and loiter from NT to $(N+1)T$. At time $(N+1)T$ when UAV_1 is done with its

tour, we show that only UAV_2 can take over successfully the mission from UAV_1. First, UAV_2 is available at time $2T + \Delta$ and can take over successfully if and only if $2T + \Delta \leq (N+1)T$, or $\Delta \leq (N-1)T$, and thus UAV_2 can do it. Next we claim that UAV_3 cannot take over successfully from UAV_1. Indeed UAV_3 can take over with no gap if and only if $3T + \Delta \leq (N+1)T$, or $\Delta \leq (N-2)T$. However, this last inequality does not hold and the claim for UAV_3 is true. It follows that besides UAV_2 none of the other UAVs are capable of taking over the mission from UAV_1 without a coverage gap. Now when UAV_2 successfully takes over the mission a similar argument shows that only UAV_3 can successfully succeed UAV_2. Repeating the same argument over several times shows that $\mathrm{UAV}_1 \overset{s}{\rightarrow} \mathrm{UAV}_2 \overset{s}{\rightarrow} \mathrm{UAV}_3 \overset{s}{\rightarrow} \cdots \overset{s}{\rightarrow} \mathrm{UAV}_N$ and when $\mathrm{UAV}_N \overset{s}{\rightarrow} \mathrm{UAV}_1$ another identical cycle starts all over again. The proof of the theorem is now complete.

2.4.2 Monotonicity Properties

The next results show the number of UAVs needed to ensure continuous coverage depends monotonically on the UAV roundtrip and the loitering times.

Corollary 2.4.2 *With the loitering time T held constant, the number of UAVs needed for continuous coverage is an increasing function of the round trip* $\Delta = \xi_1 + \xi_2 + \xi_3$.

Proof. We know from an earlier result that the optimal number of UAVs N satisfies $(N-2)T < \Delta \leq (N-1)T$. The first inequality is equivalent to $N < \Delta/T + 2$ while the second one is equivalent to $\Delta/T + 1 \leq N$. It follows by combining them that $\Delta/T + 1 \leq N < \Delta/T + 2$. Clearly, the result follows since the function $\Delta \rightarrow \Delta/T$ is an increasing

function. In fact, it is the ratio Δ/T that determines the optimal number of UAVs ensuring continuous coverage.

Corollary 2.4.3 *With the roundtrip time held constant, the number of UAVs needed for continuous coverage is a decreasing function of the loitering time* T.

Proof. Similar to the proof above since $\Delta/T + 1 \leq N < \Delta/T + 2$ and the function $T \rightarrow \Delta/T$ is decreasing.

2.5 Conclusion

In this chapter a simplified version of the UAV continuous coverage problem was studied and a formula for the optimal number of UAVs needed for continuous coverage was derived. The formula is based on the ratio of the roundtrip time to the loitering time of the UAV. This suggests that the ratio T/Δ can be used as a productivity metric of a UAV. The higher T/Δ is the better since it means the UAV is more productive by providing more loitering time with smaller support.

Chapter 3

III. THE MODEL FOR A NON-HOMOGENEOUS UAV FLEET

3.1 Introduction

The previous chapters studied the UAV continuous coverage problem with a homogeneous UAV fleet where all the UAVs had the same attribute vectors. This first approach has two advantages. First from a theoretical viewpoint it allowed us to derive a formula for the optimal cyclic schedule by just finding the minimum number of UAVs needed for continuous coverage since all of them are identical. It allows us to get basic insights and understanding of the problem that can serve as a stepping stone for the study of more complex problems. Secondly, from a practical viewpoint it may just happen, for maintenance purposes for example, that all available UAVs are of the same type in which case the model applies. A logical way of generalizing the previous model is to study the coverage problem assuming a non-homogenous UAV fleet. This approach is even more realistic because different UAVs offer unique features that can serve specific purposes. The USAF for example uses three types of UAVs; namely, the MQ-1 Predator, MQ-9 Reaper, and RQ-4 Global Hawk. These three types of UAVs have their unique characteristics and performance capabilities. Though the MQ-9 Reaper is based on the MQ-1 Predator, they do not have the same performance abilities. MQ-9 reaper has a faster cruise speed than MQ-1 Predator and it has more offensive features since it was designed to offer a striking capability by carrying up to eight Hellfire missiles. Global Hawk, with RQ-4B being its most current version of the vehicle, is primarily designed for

ISR missions. It has no striking capabilities but its high altitude and long operational radius give it a great survivability and operational flexibility. These UAV will be described in more details in a chapter devoted to some numerical applications. This chapter studies the non-homogeneous model first with deterministic and then with stochastic UAV attributes. Deterministic linear programming and chance-constrained programming are used to formulate the continuous coverage problem.

3.2 Model with Non-identical UAVs

In the basic model it is shown that an interesting relationship exists between the UAV attribute vectors and coverage that is the loitering and roundtrip times are key variables to ensure the continuous coverage. We next proceed to establish similar results when the UAVs are not necessarily identical. This model is more complex and will turned out to be a difficult combinatorial optimization problem.

3.2.1 Feasibility Condition with Non-identical UAVs

The purpose of the next result is to understand when the simplest schedule, namely a schedule of size two, is feasible. The conditions that the attributes of these two UAVs must satisfy will be extended to larger schedules. In essence, the idea expressed in the next result will be used to characterize feasibility for more general schedules. We first consider a cyclic schedule of size 2. Let (Δ_1, T_1) and (Δ_2, T_2) be the attribute vectors of

these two UAVs. Define $S_0 = 0$, $S_1 = T_1$, $S_2 = T_1 + T_2$, and $S_k = T_1 + T_2 + \cdots + T_k$, for $k \geq 1$.

Proposition 3.2.1 *Two UAVs with attribute vectors* (Δ_1, T_1) *and* (Δ_2, T_2) *form a feasible schedule if and only if* $\Delta_1 \leq T_2$ *and* $\Delta_2 \leq T_1$. *That is* $(\Delta_1, \Delta_2) \leq (T_2, T_1)$ *where the vector ordering is taken in the traditional sense of componentwise comparison.*

Proof. Consider the chart of Figure 11 which shows a size 2 schedule.

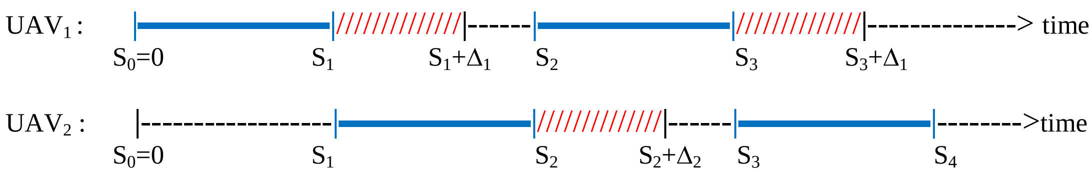


Figure 11. Cyclic Schedule with Two Non-identical UAVs

Note that the following sequence of events take place. UAV$_1$ loiters from time 0 to T_1 and is available the earliest at time $S_1 + \Delta_1$. Then UAV$_2$ loiters from time S_1 to S_2 and is available the earliest at time $S_2 + \Delta_2$. Evidently in the first cycle each UAVs can take over the mission with no coverage interruption from the previous UAV. When UAV$_2$ is done loitering at time S_2, we check whether UAV$_1$ is available to take over the mission successfully from UAV$_2$. UAV$_1$ is operationally ready the earliest at time $S_1 + \Delta_1$ and can take over the mission from UAV$_2$ without inducing a coverage gap if and only if

$$S_1 + \Delta_1 \leq S_2$$

or

$$\Delta_1 \leq T_2.$$

This inequality simply translates the fact that after departing from the target area UAV$_1$ should be able to return to it before UAV$_2$ is done with its loitering tour for otherwise a

coverage gap will be generated. Using symmetry we can immediately conclude that UAV_2 takes over the mission at S_2 from UAV_1 with no coverage gap if and only if $\Delta_2 \leq T_1$. However, we proceed to show that it is indeed the case. At time S_2, let UAV_1 take over the mission assuming that $\Delta_2 \leq T_1$. When UAV_1 is done loitering at time $S_2 + T_1$ then UAV_2 takes over at time $S_2 + T_1$ without no coverage gap if and only if it is operationally ready and available at $S_2 + T_1$; which translates to

$$S_2 + \Delta_2 \leq S_2 + T_1$$

where, once again $S_2 + \Delta_2$ is when UAV_2 is operationally ready and $S_2 + T_1$ is when UAV_1 is done with its tour. It follows from the previous inequality that UAV_2 is available to take over without coverage interruption if and only if $\Delta_2 \leq T_1$. To summarize we have the following : When UAV_2 is done, UAV_1 can take over with no coverage gap if and only if

$$\Delta_1 \leq T_2 ,$$

and when UAV_1 is done, UAV_2 can take over with no gap if and only if

$$\Delta_2 \leq T_1.$$

Therefore, the two UAVs are the exact number of UAVs needed for continuous coverage if and only if

$$\Delta_1 \leq T_2 \quad \text{and} \quad \Delta_2 \leq T_1$$

or $(\Delta_1, \Delta_2) \leq (T_2, T_1)$ as claimed.

We next search for a necessary and sufficient condition which ensures that a UAV schedule of a higher size $n > 2$ is feasible. The case of a size 2 schedule was examined previously and we found it to be feasible if and only if

$$\Delta_1 \leq T_2 \quad \text{and} \quad \Delta_2 \leq T_1.$$

We next consider 2 schedules of size 3 and 4 respectively and study their feasibility. An emerging pattern will help us formulate the feasibility conditions of a cyclic schedule and proceed to prove them in a formal way.

Case 1 : Schedule with $n = 3$ UAVs.

The Gantt chart for a cyclic schedule of size 3 is depicted in Figure 12.

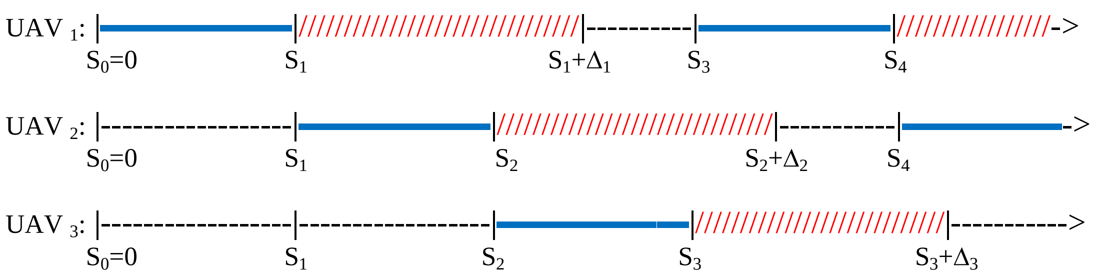


Figure 12. Feasible Schedule with 3 UAVs

From the previous chart we have the following sequence of events:

UAV$_1$: Loiters in $[S_0\,, S_1]$, is done at S_1, and is operationally ready the earliest at

$S_1 + \Delta_1$.

UAV$_2$: Loiters in $[S_1\,, S_2]$, is done at S_2, and is operationally ready the earliest at

$S_2 + \Delta_2$.

UAV$_3$: Loiters in $[S_2\,, S_3]$, is done at S_3, and is operationally ready the earliest at

$S_3 + \Delta_3$.

$\vdots$

Starting from the beginning of cycle 1 (i.e., S_3) we translate a series of successful handoffs.

UAV$_3 \overset{s}{\to}$ UAV$_1$: UAV$_1$ is operationally ready the earliest at time $S_1 + \Delta_1$ and can take over the mission with no gap if and only if $S_1 + \Delta_1 \leq S_3$ or $\Delta_1 \leq T_2 + T_3$.

UAV$_1 \overset{s}{\to}$ UAV$_2$: This can happen if and only if $S_2 + \Delta_2 \leq S_3 + T_1$ or $\Delta_2 \leq T_1 + T_3$.

UAV$_2 \overset{s}{\to}$ UAV$_3$: This can happen, if and only if $S_3 + \Delta_3 \leq S_3 + T_1 + T_2$ or $\Delta_3 \leq T_1 + T_2$.

$\vdots$

Putting the previous results together gives the following:

UAV$_3 \overset{s}{\to}$ UAV$_1$ $\Leftrightarrow$ $\Delta_1 \leq T_2 + T_3$

UAV$_1 \overset{s}{\to}$ UAV$_2$ $\Leftrightarrow$ $\Delta_2 \leq T_1 + T_3$

UAV$_2 \overset{s}{\to}$ UAV$_3$ $\Leftrightarrow$ $\Delta_3 \leq T_1 + T_2$

A pattern seems to be emerging between the roundtrip of a UAV and the loitering times of the other UAVs. To confirm that we next investigate the feasibility of a schedule of size 4.

Case 2 : Schedule with $n = 4$ UAVs.

We now consider a cyclic schedule of size 4 and search for conditions that ensure its feasibility. Using the same reasoning as in the previous case we arrive at the following statements:

$\text{UAV}_4 \overset{s}{\to} \text{UAV}_1 \iff S_1 + \Delta_1 \le S_4$

$\text{UAV}_1 \overset{s}{\to} \text{UAV}_2 \iff S_2 + \Delta_2 \le S_4 + T_1$

$\text{UAV}_2 \overset{s}{\to} \text{UAV}_3 \iff S_3 + \Delta_3 \le S_4 + T_1 + T_2$

$\text{UAV}_3 \overset{s}{\to} \text{UAV}_4 \iff S_4 + \Delta_4 \le S_4 + T_1 + T_2 + T_3$

These are equivalent to:

$\text{UAV}_4 \overset{s}{\to} \text{UAV}_1 \iff \Delta_1 \le T_4 + T_2 + T_3$

$\text{UAV}_1 \overset{s}{\to} \text{UAV}_2 \iff \Delta_2 \le T_1 + T_3 + T_4$

$\text{UAV}_2 \overset{s}{\to} \text{UAV}_3 \iff \Delta_3 \le T_1 + T_2 + T_4$

$\text{UAV}_3 \overset{s}{\to} \text{UAV}_4 \iff \Delta_4 \le T_1 + T_2 + T_3$

The four perfect handoffs of the schedule have been translated into four equivalent inequalities. A careful look at these inequalities confirms the pattern that has been observed earlier. Namely, feasibility is based on the principle that the roundtrip time of each UAV must be smaller than the aggregated loitering times of the other UAVs of the schedule.

Consider a feasible schedule of size n denoted by $A = ((\Delta_1, T_1), \dots, (\Delta_n, T_n))$. Define the scheduling cycles as follow. Let cycle 0 to be in the interval $[0, S_n]$, cycle 1 in $[S_n, 2S_n]$ and in general cycle k in $[kS_n, (k+1)S_n]$, for $k \ge 0$. The principle that has

been observed earlier by studying the feasibility of cyclic schedules of sizes 2, 3, and 4 is now formally stated and proved.

Theorem 3.2.2 *The cyclic schedule* $A = \big((\Delta_1, T_1), (\Delta_2, T_2), \dots, (\Delta_n, T_n)\big)$ *is feasible if and only if*

$$\Delta_j \leq \sum_{i=1, i \neq j}^{n} T_i \quad , \text{for } j = 1, 2, \dots, n$$

Proof. The handoff times in cycle $k \geq 1$ are given by the set $\{kS_n + S_j : j = 1,2, \dots, n\}$. Also in cycle k, for each $1 \leq j \leq n$, UAV_j is operationally available and mission ready at time $kS_n + S_j + \Delta_j$. Now in cycle k, UAV_{j-1} is done with its surveillance tour at time $kS_n + S_{j-1}$. But UAV_j in cycle $k-1$ is operationally ready at time $(k-1)S_n + S_j + \Delta_j$. Therefore, UAV_j can take over with no coverage gap if and only if

$$(k-1)S_n + S_j + \Delta_j \leq kS_n + S_{j-1}$$

or

$$\begin{aligned} \Delta_j &\leq S_n - T_j \\ &= \sum_{i=1, i \neq j}^{n} T_i. \end{aligned}$$

The next result shows that although a cyclic schedule is defined as an ordered sequence of UAVs, it turns out that as far as feasibility is concerned the order of the UAVs in a cyclic schedule does not matter. That is mainly due to the fact that feasibility depends on the relationship between the roundtrip time of a UAV and the aggregated loitering times of the remaining UAVs.

Proposition 3.2.3 *If a cyclic schedule* $A = (V_1,\ V_2,\ \ldots,\ V_n)$ *with* $V_i = (\Delta_i, T_i),\ i = 1,2,\ldots,n$ *is feasible, then any permutation of this schedule is also feasible.*

Proof. Let $(V_1,\ V_2,\ \ldots,\ V_n) = \big((\Delta_1, T_1), (\Delta_2, T_2), \ldots, (\Delta_n, T_n)\big)$ be feasible; then

$$\Delta_j + T_j \leq \sum_{i=1}^{n} T_i \equiv S_n, \quad \text{for } j = 1,2,\ldots,n.$$

Now consider a cyclic schedule $(V_{i_1},\ V_{i_2},\ \ldots,\ V_{i_n})$ where $(i_1, i_2, ,\ldots,\ i_n,)$ is a permutation of $(1,2,\ldots,n)$. Then

$$\Delta_{i_j} + T_{i_j} = \Delta_k + T_k, \ \text{ for some } k = i_j.$$

Therefore,

$$\Delta_{i_j} + T_{i_j} \leq S_n, \quad \text{for } j = 1,2,\ldots,n\,;$$

and thus $(V_{i_1},\ V_{i_2},\ \ldots,\ V_{i_n})$ is feasible.

Proposition 3.2.3 can be very helpful when solving very large scale UAV coverage problems. More specifically, as will shown later, the UAV continuous coverage problem is a combinatorial optimization problem which is formulated as a zero-one integer program (25: Sec I.2 ; 26) and is known to be computationally NP hard (4:13, 34). Therefore to find good solutions for large problems one needs to resort to heuristics. The previous proposition can help in reducing the number of size k schedules to process during a heuristic search by a factor of $k!$ because all the permutations of a schedule correspond in fact to one schedule.

3.2.2 Characterization of Minimality

A feasible cyclic schedule is graphically represented as a circuit where each node stands for a UAV and each arc for the mission handoff from a UAV to the immediate next successor. An arc here also means the handoff is successful because it does not incur a coverage gap. If a UAV can hand over the mission successfully to a UAV which is not an immediate successor then a feasible cyclic schedule of smaller size can be found and the original schedule cannot be optimal. This motivates us to introduce the "minimality" concept of a schedule.

Definition 3.2.4 *A feasible cyclic schedule S is* <u>*minimal*</u> *if no proper sub-schedule of S is feasible.*

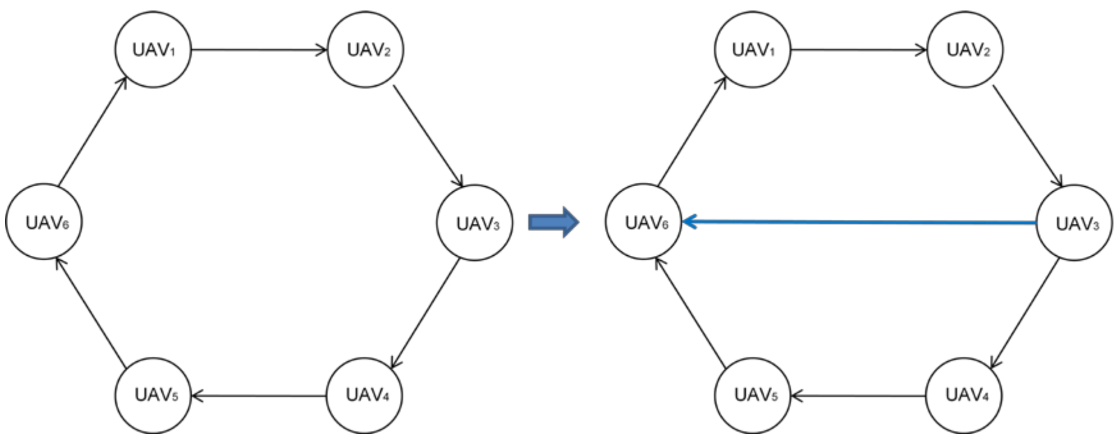


Figure 13. A Feasible but Not Minimal Schedule with 6 UAVs

Therefore a feasible schedule is minimal if it does not contain a proper circuit. Figure 13 shows a feasible cyclic schedule (UAV_1, UAV_2, UAV_3, UAV_4, UAV_5, UAV_6) which is not minimal because it contains the feasible cyclic schedule (UAV_1, UAV_2, UAV_3, UAV_6). Note also that a minimal cyclic schedule is not the same as a minimum cyclic

schedule because as minimum schedule is necessarily minimal but not the other way around.

For the purpose of deriving the minimality conditions of an arbitrary feasible cycle we first investigate those conditions for a small schedule of size 6 of the form $S = (V_1, V_2, \dots V_6)$ where $V_i = (\Delta_i, T_i)$, $i = 1,2,\dots,6$. The conditions obtained for this schedule will be generalized to any feasible cyclic schedule.

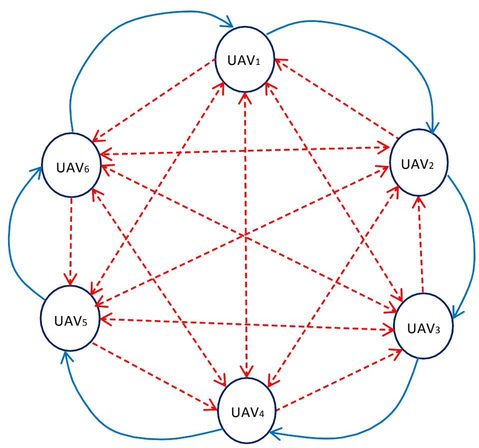


Figure 14. A Minimal Feasible Cyclic Schedule

In Figure 14, the feasibility of S is represented by the solid handoff arcs and the minimality of S is represented by the fact that the dotted handoff arcs should not take place. In the cyclic schedule $S = (V_1, V_2, \dots, V_6)$ the UAVs need to loiter and relay each other in an orderly fashion as dictated by the cyclic schedule for them to ensure continuous coverage. We proceed to translate the handoff arcs in Figure 14 for each UAV. Each UAV gives rise to a set of inequalities. First, consider the handoff arcs which stem from UAV_1.

From the feasibility criterion seen earlier in order for UAV_2 to have the capability to take over the mission successfully from UAV_1 its roundtrip Δ_2 must satisfy :

$$\mathrm{UAV}_1 \xrightarrow{s} \mathrm{UAV}_2 \quad \Leftrightarrow \quad \Delta_2 \leq T_1 + T_3 + T_4 + T_5 + T_6.$$

To ensure minimality UAV_1 should not have the capability to bypass UAV_2 and hand over the mission to UAV_3 successfully. To clarify that consider the following chart.

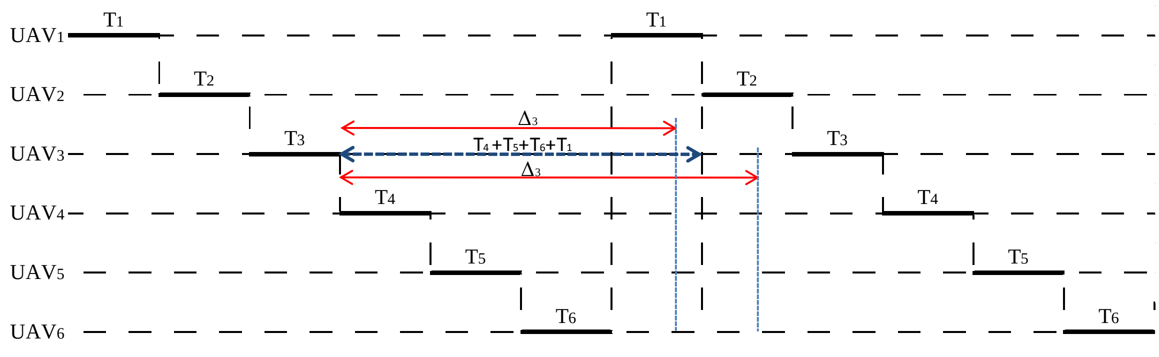


Figure 15. Minimality Condition for UAV_3 Roundtrip Time

When UAV_1 finishes its loitering time it is required that UAV_3 should not be able to take over the mission form UAV_1. In other words $\Delta_3 \leq T_4 + T_5 + T_6 + T_1$ should not hold and so the following inequality is needed for the minimality of the schedule.

$$\mathrm{UAV}_1 \xrightarrow{s} \mathrm{UAV}_3 \quad \Leftrightarrow \quad T_4 + T_5 + T_6 + T_1 < \Delta_3.$$

The other minimality conditions which are based on the principle that a UAV should not be able to handover the mission to any other UAV but its immediate successor in the cyclic schedule are derived in a similar fashion. They are :

$$\mathrm{UAV}_1 \xrightarrow{\not s} \mathrm{UAV}_4 \quad \Leftrightarrow \quad T_5 + T_6 + T_1 < \Delta_4.$$

$$\mathrm{UAV}_1 \xrightarrow{\not s} \mathrm{UAV}_5 \quad \Leftrightarrow \quad T_6 + T_1 < \Delta_5,$$

$$\mathrm{UAV}_1 \xrightarrow{\not s} \mathrm{UAV}_6 \quad \Leftrightarrow \quad T_1 < \Delta_6.$$

Using the same principle as before each of the remaining UAVs will generate a group of feasibility and minimality conditions as follows.

UAV₂

$\mathrm{UAV}_2 \overset{s}{\rightarrow} \mathrm{UAV}_3 \iff \Delta_3 \leq T_1 + T_2 + T_4 + T_5 + T_6$

$\mathrm{UAV}_2 \overset{\not s}{\rightarrow} \mathrm{UAV}_4 \iff T_5 + T_6 + T_1 + T_2 < \Delta_4$

$\mathrm{UAV}_2 \overset{\not s}{\rightarrow} \mathrm{UAV}_5 \iff T_6 + T_1 + T_2 < \Delta_5$

$\mathrm{UAV}_2 \overset{\not s}{\rightarrow} \mathrm{UAV}_6 \iff T_1 + T_2 < \Delta_6$

$\mathrm{UAV}_2 \overset{\not s}{\rightarrow} \mathrm{UAV}_1 \iff T_2 < \Delta_1$

UAV_3

$\mathrm{UAV}_3 \overset{s}{\rightarrow} \mathrm{UAV}_4 \iff \Delta_4 \leq T_1 + T_2 + T_3 + T_5 + T_6$

$\mathrm{UAV}_3 \overset{\not s}{\rightarrow} \mathrm{UAV}_5 \iff T_6 + T_1 + T_2 + T_3 < \Delta_5$

$\mathrm{UAV}_3 \overset{\not s}{\rightarrow} \mathrm{UAV}_6 \iff T_1 + T_2 + T_3 < \Delta_6$

$\mathrm{UAV}_3 \overset{\not s}{\rightarrow} \mathrm{UAV}_1 \iff T_2 + T_3 < \Delta_1$

$\mathrm{UAV}_3 \overset{\not s}{\rightarrow} \mathrm{UAV}_2 \iff T_3 < \Delta_2$

UAV_4

$\mathrm{UAV}_4 \overset{s}{\rightarrow} \mathrm{UAV}_5 \iff \Delta_5 \leq T_1 + T_2 + T_3 + T_4 + T_6$

$\mathrm{UAV}_4 \overset{\not s}{\rightarrow} \mathrm{UAV}_6 \iff T_1 + T_2 + T_3 + T_4 < \Delta_6$

$\mathrm{UAV}_4 \overset{\not s}{\rightarrow} \mathrm{UAV}_1 \iff T_2 + T_3 + T_4 < \Delta_1$

$\mathrm{UAV}_4 \overset{\not s}{\rightarrow} \mathrm{UAV}_2 \iff T_3 + T_4 < \Delta_2$

$\mathrm{UAV}_4 \overset{\not s}{\rightarrow} \mathrm{UAV}_3 \iff T_4 < \Delta_3$

UAV_5

$\mathrm{UAV}_5 \overset{s}{\rightarrow} \mathrm{UAV}_6 \iff \Delta_5 \leq T_1 + T_2 + T_3 + T_4 + T_5$

$\mathrm{UAV}_5 \overset{\not s}{\rightarrow} \mathrm{UAV}_1 \iff T_2 + T_3 + T_4 + T_5 < \Delta_1$

$\mathrm{UAV}_5 \overset{\not s}{\rightarrow} \mathrm{UAV}_2 \iff T_3 + T_4 + T_5 < \Delta_2$

$\mathrm{UAV}_5 \overset{\not s}{\rightarrow} \mathrm{UAV}_3 \iff T_4 + T_5 < \Delta_3$

$\mathrm{UAV}_5 \overset{\not s}{\rightarrow} \mathrm{UAV}_4 \iff T_5 < \Delta_4$

UAV_6

$\mathrm{UAV}_6 \overset{s}{\rightarrow} \mathrm{UAV}_1 \iff \Delta_1 \leq T_2 + T_3 + T_4 + T_5 + T_6$

$$\mathrm{UAV}_6 \overset{\nrightarrow}{\rightarrow} \mathrm{UAV}_2 \quad \Leftrightarrow \quad T_3 + T_4 + T_5 + T_6 < \Delta_2$$
$$\mathrm{UAV}_6 \overset{\nrightarrow}{\rightarrow} \mathrm{UAV}_3 \quad \Leftrightarrow \quad T_4 + T_5 + T_6 < \Delta_3$$
$$\mathrm{UAV}_6 \overset{\nrightarrow}{\rightarrow} \mathrm{UAV}_4 \quad \Leftrightarrow \quad T_5 + T_6 < \Delta_4$$
$$\mathrm{UAV}_6 \overset{\nrightarrow}{\rightarrow} \mathrm{UAV}_5 \quad \Leftrightarrow \quad T_6 < \Delta_5$$

From the above inequalities, the feasibility part gives:

$$\Delta_j \le \sum_{i=1, i \neq j}^{6} T_i \text{ for } j = 1, 2, \dots, 6$$

which is as given in Theorem 3.2.2 when $n = 6$. Next, we collect those inequalities involving the roundtrip durations to establish the conditions that ensure minimality of this schedule.

Inequality for Δ_1 : $T_2 + T_3 + T_4 + T_5 < \Delta_1$,

Inequality for Δ_2 : $T_3 + T_4 + T_5 + T_6 < \Delta_2$,

Inequality for Δ_3 : $T_1 + T_4 + T_5 + T_6 < \Delta_3$,

Inequality for Δ_4 : $T_1 + T_2 + T_5 + T_6 < \Delta_4$,

Inequality for Δ_5 : $T_1 + T_2 + T_3 + T_6 < \Delta_5$,

Inequality for Δ_6 : $T_1 + T_2 + T_3 + T_4 < \Delta_6$.

It follows that the round trip times need to satisfy the following inequalities for the cyclic schedule to be feasible and minimal.

$$T_2 + T_3 + T_4 + T_5 < \Delta_1 \le T_2 + T_3 + T_4 + T_5 + T_6$$
$$T_3 + T_4 + T_5 + T_6 < \Delta_2 \le T_1 + T_3 + T_4 + T_5 + T_6$$
$$T_1 + T_4 + T_5 + T_6 < \Delta_3 \le T_1 + T_2 + T_4 + T_5 + T_6$$
$$T_1 + T_2 + T_5 + T_6 < \Delta_4 \le T_1 + T_2 + T_3 + T_5 + T_6$$

$$T_1 + T_2 + T_3 + T_6 < \Delta_5 \leq T_1 + T_2 + T_3 + T_4 + T_6$$

$$T_1 + T_2 + T_3 + T_4 < \Delta_6 \leq T_1 + T_2 + T_3 + T_4 + T_5$$

It follows that, in general, in order to obtain the feasibility and minimality conditions of the cyclic schedule $S = ((\Delta_1, T_1), (\Delta_2, T_2), \dots, (\Delta_n, T_n))$ we must have for each $i = 2, \dots, n$;

$$T_1 + \cdots + T_{i-2} + T_{i+1} + \cdots + T_n < \Delta_i \leq T_1 + \cdots + T_{i-1} + T_{i+1} + \cdots + T_n$$

and for $i = 1$,

$$T_2 + T_3 + \cdots + T_{n-1} < \Delta_1 \leq T_2 + T_3 + \cdots + T_n.$$

This result is stated next and proved formally.

Theorem 3.2.5 *Let $A = (V_1, V_2, \dots, V_n)$ be a cyclic schedule where $V_i = (\Delta_i, T_i)$ for $i = 1,2, \dots, n$. Define by $T_0 = T_n$ and $S_k = T_1 + T_2 + \cdots + T_k, 1 \leq k \leq n$. Then, A is feasible and minimal if and only if*

$$S_n - T_{j-1} < T_j + \Delta_j \leq S_n, \quad j = 1,2, \dots, n \tag{3.2.1}$$

Before proving this result note that (3.2.1) is equivalent to the following two groups of inequalities:

$$T_j + \Delta_j \leq \sum_{i=1}^{n} T_i \quad \text{for } j = 1,2, \dots, n, \tag{3.2.2}$$

and

$$\sum_{i=1}^{n} T_i - T_{j-1} < T_j + \Delta_j \quad \text{for } j = 1,2, \dots, n. \tag{3.2.3}$$

It is easy to see that (3.2.2) is equivalent to

$$\Delta_j \leq \sum_{i=1}^{n} T_i - T_j$$

$$= \sum_{i=1, i \neq j}^{n} T_i \quad , j = 1,2, \dots , n.$$

But these are just the feasibility inequalities that have been proved earlier. Note that inequalities (3.2.3) are equivalent to :

$$\sum_{i=1}^{n} T_i - T_{j-1} < T_j + \Delta_j \quad , j = 1,2, \dots , n$$

or

$$\sum_{i=1, i \neq j}^{n} T_i - T_{j-1} < \Delta_j \quad , j = 1,2, \dots , n.$$

As separate the cases $j = 1$ and $j \geq 2$, they can be written as:

$$\sum_{i=1, i \neq j, i \neq j-1}^{n} T_i < \Delta_j \quad , j = 2,3, \dots , n$$

and

$$\sum_{i=2}^{n-1} T_i < \Delta_j \quad , j = 1.$$

Therefore, for minimality we need to show the following inequalities :

$$\sum_{i=1, i \neq j, i \neq j-1}^{n} T_i < \Delta_j \quad , j = 2,3, \dots , n \tag{3.2.4}$$

and

$$\sum_{i=2}^{n-1} T_i < \Delta_j \quad , j = 1. \tag{3.2.5}$$

Proof of Theorem 3.2.6. The feasibility was proved earlier. We focus on the minimality. Let us draw a picture which in fact is an extract from the Gantt chart of the UAV cyclic schedule of size n because it focuses on cycles k and $k+1$ of the schedule.

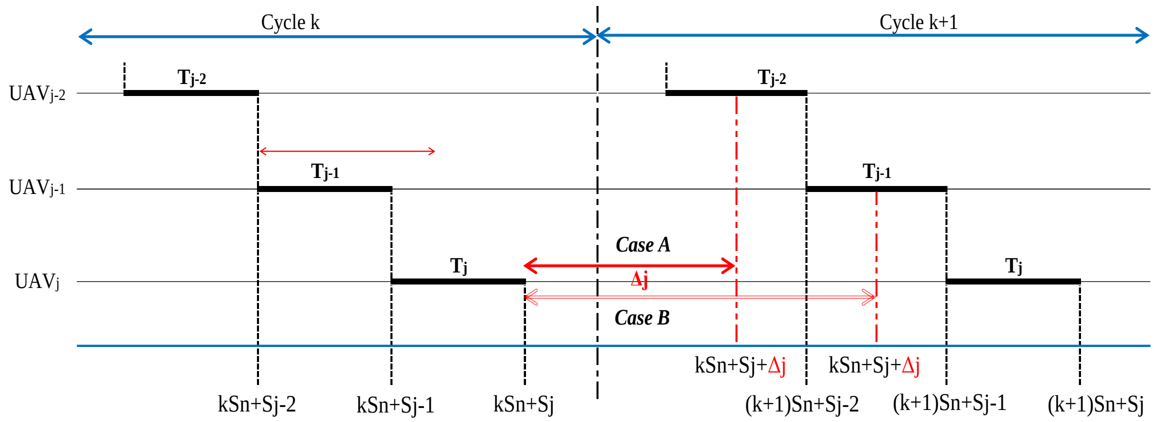


Figure 16. Cycle k and k + 1 of The Schedule of Size n

This picture shows that UAV_{j-1} is done loitering at time $(k+1)S_n + S_{j-1}$ and UAV_{j-1} is ready to pass on the mission to the next UAV_j at that time. UAV_j can make it to the target area to take over the mission from UAV_{j-1} because the schedule is feasible. The earliest time that UAV_j is available to take over loitering is at time $kS_n + S_j + \Delta_j$. We know from feasibility that

$$kS_n + S_j + \Delta_j \leq (k+1)S_n + S_{j-1}.$$

In order to ensure minimality we should make sure that UAV_j does not have the capability to be available and mission ready before UAV_{j-1} does, for otherwise there is no

need to have UAV_{j-1} in the schedule. In Figure 16, we represented the time $kS_n + S_j + \Delta_j$ for two possible cases denoted by (A) and (B). The time $kS_n + S_j + \Delta_j$ is always less than or equal to $(k+1)S_n + S_{j-1}$ because of feasibility. Both cases (A) and (B) satisfy that ; to ensure minimality we basically avoid the occurance of the situation depicted in case (B). Namely, UAV_j should not be available before time $(k+1)S_n + S_{j-1}$ for otherwise UAV_{j-1} would not be needed. Thus, we must have :

$$(k+1)S_n + S_{j-2} < kS_n + S_j + \Delta_j,$$

$$S_n - \left(S_j - S_{j-2}\right) < \Delta_j,$$

$$S_n - \left(T_j - T_{j-1}\right) < \Delta_j \quad , j = 1,2, \dots, n,$$

or

$$\sum_{i=1, i \neq j, i \neq j-1}^{n} T_i < \Delta_j \quad , j = 1,2, \dots, n.$$

Indeed, if the above inequality holds in the opposite direction, that is if

$$kS_n + S_j + \Delta_j \leq (k+1)S_n + S_{j-2},$$

then, in cycle $(k+1)$, UAV_{j-1} can be skipped by “switching” from UAV_{j-2} to UAV_j and then the schedule will not be minimal. For the case $j = 1$, since UAV_1 follows UAV_n (i.e. UAV_n preceeds UAV_1) in the schedule we must have

$$S_n - (T_n - T_1) < \Delta_1$$

or

$$\sum_{i=2}^{n-1} T_i < \Delta_1.$$

The proof is now complete.

3.2.3 Handling Small Coverage Gaps

So far the model we considered assumed that handoffs between UAVs are perfect in the sense that no coverage interruption takes place. In practice, however, it may be the case that a relatively small coverage gap may be acceptable because it is deemed not to affect the mission objective and requirements. This leads to us to introduce an extension of the previous model that takes into account the fact an ISR mission can tolerate a small coverage gap in a typical UAV handoff. The following charts shows a handoff between two UAVs with a coverage gap of length ε.

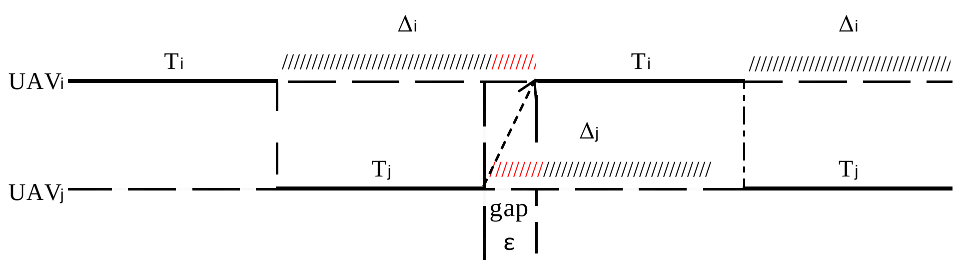


Figure 17. Coverage Gap of Length ε

Consider a typical perfect handoff from UAV_i to UAV_j in a cyclic schedule. The handoff from UAV_i to UAV_j is perfect because $\Delta_j - T_i \leq 0$. Next consider the situation where the mission can tolerate a small coverage gap of some length $\varepsilon > 0$. This means that we now consider a handoff to be successful if $\Delta_j - T_i \leq \varepsilon$. Note that $\Delta_j - T_i \leq \varepsilon$ is the sense as $\Delta_j \leq T_i + \varepsilon$ and therefore by defining $T_i' = T_i + \varepsilon$ for all i we can switch

from UAV_i to UAV_j with a gap of maximum length ε if and only if $\Delta_j \leq T_i'$. From this observation, the original cyclic schedule $S = \big((\Delta_1, T_1), (\Delta_2, T_2), \dots, (\Delta_n, T_n)\big)$ can be transformed into a new cyclic schedule $S' = \big((\Delta_1, T_1{}'), (\Delta_2, T_2{}'), \dots, (\Delta_n, T_n{}')\big)$ which considers any positive gap in a handoff as a successful handoff and therefore we back to the early setting. Basically, the earlier definition of a perfect handoff is equivalent to a handoff with a coverage gap of length $\varepsilon = 0$ (i.e., null coverage gap). In fact, when a schedule can tolerate a coverage gap of a most ε it is same as adding a loitering capability of ε time units to the UAVs. It follows that set of all cyclic schedules for which a coverage gap of at most ε is admissible is much larger than the set of all schedules with perfect handoffs. Therefore an optimal cyclic schedule which tolerates a positive gap will have a smaller number of UAVs than an optimal schedule with perfect handoffs. This can be easily seen in the particular case of a homogeneous fleet by invoking the main theorem of the basic model. Since the optimal number of UAVs is basically determined by the ratio Δ / T, it follows that $\Delta / (T + \varepsilon) < \Delta / T$ implies that an optimal cyclic schedule will need a smaller number of UAVs when a coverage gap of maximum length ε is acceptable. It follows from Theorem 3.4.3 that the new cyclic schedule $S' = ((\Delta_1, T_1'), (\Delta_2, T_2') \ \dots, (\Delta_n, T_n'))$ is feasible if and only if

$$T_j' + \Delta_j \leq \sum_{i=1}^{n} T_i' \ \text{ for } j = 1,2, \dots, n$$

or

$$\Delta_j \ \leq \ (n-1)\varepsilon \ + \sum_{i=1, i \neq j}^{n} T_i \ \text{ for } j = 1,2, \dots, n$$

This shows that for the new cyclic schedule to be feasible each $\mathrm{UAV_j}$ can be away from the target area for an additional period not exceeding $(n-1)\varepsilon$..

3.3 Linear Programming Formulation

In this section we use linear programming to find an optimal cyclic schedule that ensures continuous coverage of the target area. To formulate the UAV scheduling problem as a linear program we need to define the decision variables, objective function, and constraints (26).

Recall that the fleet F is written as $F = \{(\Delta_1, T_1), (\Delta_2, T_2), \dots, (\Delta_K, T_K)\}$. First, we define the binary decision variables x_j according to whether $\mathrm{UAV_j}$ is included in the cyclic schedule or not.

$$x_j = \begin{cases} 1 & \text{if } \mathrm{UAV_j} \text{ is included in the schedule} \\ 0 & \text{if } \mathrm{UAV_j} \text{ is not included in the schedule} \end{cases}$$

Since the purpose is to find the minimum number of UAVs that provides continuous coverage the objective function of the linear program is simply the sum of the decision variables. Thus the objective function is :

$$\min \;\; z = x_1 + x_2 + \dots + x_K.$$

The feasible region of the continuous coverage linear program corresponds to the set of all feasible cyclic schedules. But the latter set is characterized by Theorem 3.2.2 the results of which translate exactly into the constraints of the linear program sought after. For convenience, let

$$a_i = T_i \text{ and } b_i = \Delta_i \quad , i = 1,2, \dots, K.$$

Then using Theorem 3.2.2 the feasibility conditions of a UAV cyclic schedule are:

$$\sum_{j=1, j \neq i}^{N} a_j x_j \geq b_i \quad , i = 1,2, \dots, K.$$

Therefore, the optimal cyclic schedule is a solution to the following binary integer programming problem:

$$\min\ z = \sum_{i=1}^{K} x_i$$

subject to

$$\sum_{\substack{j=1 \\ j \neq i}}^{N} a_j x_j \geq b_i \quad , i = 1,2, \dots, K$$

$$x_i = \{0, 1\}^K \quad , \text{all } i \in \{1,2, \dots, K\}$$

Define the $K \times K$ square matrix $A = [a_{ij}]$ as follows. For each $i = 1, 2, \dots, K$, and $j = 1, 2, \dots, K$, let

$$a_{ij} = \begin{cases} a_j & , \text{for } j = 1,2, \dots, K, j \neq i \\ 0 & , \text{for } j = i \end{cases}$$

Next define

$$x = (x_1, x_2, \dots, x_K)^T, b = (b_1, b_2, \dots, b_K)^T, \text{ and } e = (1, 1, \dots, 1),$$

where the last row vector e has K components.

Then the binary integer linear programming can be rewritten as:

$$\min \ z = ex$$

subject to

$$Ax \geq b$$

$$x \in \{0,1\}^K$$

Although we do not exploit the special structure of the matrix A it is worth mentioning that

$$A = \begin{bmatrix} 0 & a_2 & a_3 & & a_{K-1} & a_K \\ a_1 & 0 & a_3 & \cdots & a_{K-1} & a_K \\ a_1 & a_2 & 0 & & a_{K-1} & a_K \\ & \vdots & & \ddots & \vdots & \\ a_1 & a_2 & a_3 & \cdots & 0 & a_K \\ a_1 & a_2 & a_3 & & a_{K-1} & 0 \end{bmatrix}$$

is a non-negative matrix. All entries of a column are the same except for the entry on the main diagonal which is null. Its determinant is

$$\det(A) = (-1)^{K-1}(K-1)\prod_{i=1}^{K} a_k.$$

Than can be seen as follows. First by using row permutations we can move the last row to the top to have

$$\det(A) = (-1)^{K-1}\det\begin{bmatrix} a_1 & a_2 & a_3 & & a_{K-1} & 0 \\ 0 & a_2 & a_3 & \cdots & a_{K-1} & a_K \\ a_1 & 0 & a_3 & & a_{K-1} & a_K \\ & \vdots & & \ddots & \vdots & \\ a_1 & a_2 & a_3 & \cdots & a_{K-1} & a_K \\ a_1 & a_2 & a_3 & & 0 & a_K \end{bmatrix}.$$

Then using Gauss elimination we get

$$\det(A) = (-1)^{K-1}\det\begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{K-2} & a_{K-1} & 0 \\ 0 & a_2 & a_3 & \cdots & a_{K-2} & a_{K-1} & a_K \\ 0 & 0 & a_3 & \cdots & a_{K-2} & a_{K-1} & 2a_K \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & a_{K-2} & a_{K-1} & (K-3)a_K \\ 0 & 0 & 0 & \cdots & 0 & a_{K-1} & (K-2)a_K \\ 0 & 0 & 0 & \cdots & 0 & 0 & (K-1)a_K \end{bmatrix}$$

and the result follows. Since all of the loitering times $a_k, 1 \le k \le K$ are positive numbers it follows that the determinant is not null and A is invertible.

In this section the UAV continuous coverage problem is formulated as a binary integer linear program model with a minimizing objective function. It follows that for a large UAV fleet this model is a computationally NP hard problem because the zero-one integer linear programming problem is known to be NP hard (25: Sec I.1). Clearly, with a UAV fleet of a small size this model can be solved within a reasonable time by traditional techniques such as the branch and bound method, but as the UAV fleet size increases this branch and bound technique is not efficient enough to find an optimal solution because it might take a relatively long running time and may even fail to find an optimal solution. Thus, a heuristic algorithmic approach (e.g., Tabu search) may be needed to solve the model.

3.4 Stochastic Model

We now extend the UAV deterministic model to a stochastic model with non-identical UAVs. We assume that we have a non-homogeneous UAV fleet. In practice, loitering and roundtrip times are random because of weather conditions, enemy hostility, payload weight, etc., and imperfect handoffs may occur with a positive probability. As a result,

we assume that Δ and T are non-negative random variables. Coverage now becomes probabilistic and the constraints of the binary integer program derived earlier should reflect that by putting them in the form:

$$P(\text{Continuous Coverage}) \geq \text{threshold probability.}$$

For example we may be interested in finding the best UAV cycle schedule that provides continuous coverage 90% of the time by having a constraint of the form

$$P(\text{Continuous Coverage}) \geq 90\%.$$

This leads in a natural way to formulate the stochastic version of the UAV continuous coverage problem as a chance-constrained programming problem (27: 23-27). This may be regarded as a "risk sensitive" formulation of the coverage constraints. For comparison purposes we may also formulate the coverage constraints using a "risk-neutral" approach by taking the expected value of the coverage constraints.

3.4.1 Risk Neutral Model

The constraints of the deterministic linear programming formulation for the UAV continuous coverage problem are

$$\sum_{\substack{j=1 \\ j \neq i}}^{K} a_j x_j \geq b_i, \quad i = 1,2, \dots, K.$$

The risk neutral approach consists in taking for each $i \in \{1, 2, \dots, K\}$ the expected value on both sides of the above constraints. This means that on the average, the roundtrip of UAV_i is smaller than the aggregated random loitering times of the other UAVs in the

cyclic schedule. The decision variables are always deterministic since they translate whether or not a UAV is included in the schedule. Taking the expected values on both sides of the previous constraints yields :

$$\sum_{\substack{j=1 \\ j\neq i}}^{K} E\left[a_j\right]x_j \geq E[b_i], \quad i = 1,2,\dots,K$$

or

$$\sum_{\substack{j=1 \\ j\neq i}}^{K} E\left[a_j\right]x_j \geq \mu_i, \quad i = 1,2,\dots,K$$

where $\mu_i = E[b_i]$.

The risk neutral approach to the stochastic coverage problem is based on the idea of replacing each random variable by its mean. Therefore, when the distribution functions of the roundtrip and loitering times are known, the means are also known, and a deterministic model is easily obtained. The linear programming model that results through this process is :

$$\min\ z = \sum_{i=1}^{K} x_i$$

subject to

$$\sum_{\substack{j=1 \\ j\neq i}}^{K} E\left[a_j\right]x_j \geq \mu_i, \qquad i \in \{1,2,\dots,K\}$$

$$x_i = \{0,1\}^K, \quad \text{all } i \in \{1,2,\dots,K\}$$

The previous approach was based on the risk neutrality (indifference) of the decision maker by using the expected value metric. The mean value of a random variable is just a measure of location and does not say much about the spread of the values of the random variable around the mean and this can be unacceptable to the decision maker unless he is insensitive to risk. However, if the decision maker cares about risk he may choose an appropriate metric to control it. The next section introduces one approach to controlling the risk by imposing a lower bound on the continuous coverage probability.

3.4.2 Risk Sensitive Model

Another approach to handle the stochastic coverage problem is through chance-constrained programming and the model obtained is this fashion is referred to the as the risk sensitive model. Now the parameters a_j and b_j, $1 \leq j \leq K$ are non-negative random variables and each coverage constraint takes the general form of

$$P(\text{Continuous Coverage}) \geq \text{threshold probability}.$$

More specifically, by using a threshold probability $\alpha_i, 0 \leq \alpha_i \leq 1$ we can express each coverage constraint as

$$P\left(\sum_{j=1, j\neq i}^{K} a_j x_j \geq b_i\right) \geq \alpha_i \quad , i = 1,2, \dots, K$$

$$x_i = \{0, 1\}^K \quad , \text{all } i \in \{1,2, \dots, K\}$$

where α_i denotes a lower bound to the probability of satisfying the i^{th} constraint. A special case worth mentioning is when the threshold probabilities α_i are all equal to

some common value α. Note that the decision variable x_i is still binary and the objective function is also deterministic. The chance constrained programming for this UAV scheduling problem is:

$$\min \ z = \sum_{i=1}^{K} x_i$$

subject to

$$P\left(\sum_{j=1, j\neq i}^{K} a_j x_j \geq b_i\right) \geq \alpha_i$$

$$x_i = \{0, 1\}^K$$

$$0 \leq \alpha_i \leq 1, \quad \text{all } i \in \{1,2,\ldots,K\}$$

where the a_j and b_j parameters are non-negative random variables as mentioned earlier. Next, assume that the probability distribution function of each b_j is normal and the parameters a_j are constant. Based on these assumptions we can convert this chance constrained programming model into a linear programming model by using properties of normal distributions.

For each $i \in \{1,2,\ldots,K\}$ let b_i be normally distributed with mean μ_i and standard deviation σ_i. Then we have

$$P\left(\sum_{j=1, j\neq i}^{K} a_j x_j \geq b_i\right) = P\left(\frac{\sum_{j=1,j\neq i}^{K} a_j x_j - \mu_i}{\sigma_i} \geq \frac{b_i - \mu_i}{\sigma_i}\right)$$

where $(b_i - \mu_i)/\sigma_i$ has the standard normal distribution. Using the CDF table of the standard normal distribution we define K_{α_i} to be the constant that satisfies:

$$P\left(Z \le K_{\alpha_i}\right) = \alpha_i$$

where $Z \sim N(0,1)$ and α_i is a number between 0 and 1. Thus

$$P\left(\frac{b_i - \mu_i}{\sigma_i} \le K_{\alpha_i}\right) = \alpha_i. \tag{3.4.1}$$

Because the goal is to convert the chance-constrained program model into a linear programming model we write the constraints as :

$$P\left\{\frac{b_i - \mu_i}{\sigma_i} \le \frac{\sum_{j=1,j\neq i}^{K} a_j x_j - \mu_i}{\sigma_i}\right\} \ge \alpha_i \quad , i = 1,2,\dots,K. \tag{3.4.2}$$

Note that K_{α_i} should be smaller than $\left(\sum_{j=1,j\neq i}^{N} a_j x_j - \mu_i\right)/\sigma_i$ to satisfy the (3.4.1) and (3.4.2). As a result, the constraints

$$P\left(\sum_{j=1,j\neq i}^{K} a_j x_j \ge b_i\right) \ge \alpha_i, \quad i = 1,2,\dots,K$$

are equivalent to

$$\frac{\sum_{j=1,j\neq i}^{K} a_j x_j - \mu_i}{\sigma_i} \ge K_{\alpha_i} \quad , i = 1,2,\dots,K$$

or

$$\sum_{j=1,j\neq i}^{K} a_j x_j \ge \mu_i + K_{\alpha_i}\sigma_i \quad , i = 1,2,\dots,K.$$

Thus, the probability constraints are converted to linear constraints. Since the linear constraints are equivalent to the probabilistic constraints the previous chance constrained program is reduced to the following equivalent linear program:

$$\min\ z = \sum_{i=1}^{K} x_i$$

subject to

$$\sum_{j=1, j\neq i}^{K} a_j x_j \geq \mu_i + K_{\alpha_i}\sigma_i, \qquad i \in \{1,2,\dots,K\}$$

$$x_i = \{0, 1\}, \quad \mathrm{b_i} \sim N(\mu_i, \sigma_i)$$

$$0 \leq \alpha_i \leq 1, \quad \text{all } i \in \{1,2,\dots,K\}.$$

3.5 Conclusion

In this chapter we studied the UAV continuous coverage problem for a non-homogeneous UAV fleet. We formulated the UAV coverage problem as a zero-one integer linear program and introduced the notion of a minimal UAV schedule which can be used when solving large scale UAV continuous coverage models using heuristics. Finally the stochastic coverage problem was formulated as a chance-constrained problem that we converted to a zero-one integer linear program. Numerical examples will provided in the next chapter.

Chapter 4

IV. NUMERICAL APPLICATIONS

4.1 Introduction

In the previous chapters we established a modeling framework for the UAV continuous coverage problem and derived some theoretical as well as practical results. The ultimate goal of a model is the applicability of its insights and results to practical problems. A model is most useful when it can be applied to practical situations and has the potential to improve performance and add value to the current state of affairs. Because it is the first time that such a UAV scheduling modeling approach has been attempted practical applications with potential benefits to the war fighter may still need to wait until further results are established. A great deal of efforts were spent to find the right mathematical approach to the problem and as a consequence valuable applications may need have to wait until more realistic extensions of the model are added. Nevertheless in this chapter we attempt to provide some simple examples to illustrate the formulations of the UAV continuous coverage problem that we have established

We speculate that the USAF has a classified process for scheduling UAVs to conduct ISR missions and we have no way of relating the present efforts to that. Knowing how the USAF does its scheduling for continuous coverage may lead to new ideas and possibly new directions for more operationally relevant research.

The next sections develop some simple numerical applications using the USAF UAVs that are currently in use in various part of the world. These include the MQ-1 Predator, MQ-9 Reaper, and RQ-4 Global Hawk (17:8,10).

USAF describes the MQ-1 Predator UAV as a medium-altitude and long-endurance UAV system. It can serve in surveillance and reconnaissance roles and fire two Hellfire missiles, and the aircraft, in use since 1995, has seen combat over Afghanistan, Pakistan, Bosnia, Serbia, Iraq, and Yemen (28, 29).

Figure 18. MQ-1 Predator

Table 1. MQ-1 Predator General Characteristics

Empty Weight	512 kg	Maximum Speed	135 mph
Loaded Weight	1,020 kg	Cruise Speed	81-103 mph
Max Takeoff Weight	1,020 kg	Stall Speed	62 mph
Service Ceiling	25,000 ft	Range	2,000 nm
Endurance	24 hrs		

MQ-9 Reaper is developed for use by the United States Air Force, the United States Navy, Italian Air Force, and the Royal Air Force. The MQ-9 is the first hunter-killer UAV designed for long-endurance, high-altitude surveillance (17:63; 24).

Table 2. MQ-9 Reaper General Characteristics

Empty Weight	2,223 kg	Maximum Speed	300 mph
Loaded Weight	4,760 kg	Cruise Speed	172-195 mph
Max Takeoff Weight	4,760 kg	Endurance	24 hrs
Service Ceiling	50,000 ft	Range	1,655 nm

Figure 19. MQ-9 Reaper

RQ-4 Global Hawk is the fastest UAV flying today, it can provide a broad overview and systematically target surveillance shortfalls at long range with long loitering times over target areas. Also, it can survey as much as 40,000 square miles (100,000 square kilometers) of terrain a day (24, 25).

Table 3. RQ-4 Global Hawk General Characteristics

Empty Weight	3,851 kg	Maximum Speed	454 mph
Loaded Weight	10,387 kg	Cruise Speed	404 mph
Max Takeoff Weight	10,387 kg	Endurance	42 hrs
Service Ceiling	65,000 ft		

Figure 20. RQ-4 Global Hawk

In this chapter, we study UAV continuous coverage models using some UAV fleets which consist of these three kinds of UAVs. The previous results are used to find the optimal UAV number for each UAV fleet and target areas. Deterministic linear programming is used first and the stochastic model follows.

4.2 Deterministic Modeling Applications

The UAV fleet we consider in this section consists of 20 UAVs − 8 MQ-1 Predator's, 8 MQ-9 Reaper's, and 4 RQ-4 Global Hawk's. The target area is 1,100 miles away from

the operating base. Each UAV needs 5 hours for maintenance and refueling and their cruise speeds are obtained from the average of the cruise speed range. Based on these assumptions the loitering and roundtrip times can be calculated. The following table displays the results of the calculations.

Table 4. Roundtrip and Loitering Times (hours) : $d = 1{,}100$ Miles

UAVs	ξ_1	ξ_2	ξ_3	*Roundtrip time*	*Loitering time*
MQ-1 Predator	12.0	12.0	5	28.9	0.087
MQ-9 Reaper	6.0	6.0	5	17.0	12.0
RQ-4 Global Hawk	2.7	2.7	5	10.4	36.6

The values of ξ_1 and ξ_2 have been calculated based on the UAV speed and distance between the operating base and target area. Adding these two values to the maintenance and refueling time (5 hours) gives the roundtrip of the UAV. The UAV loitering time is obtained by subtracting the back and forth trip durations from the UAV endurance time.

In this model there are 20 binary integer decision variables. The matrix A is

$$A = \begin{bmatrix} 0 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 36.554 \\ 0.087 & 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 & 36.554 & \cdots & 0 \end{bmatrix}$$

and the vectors b and x are:

$$b = [28.913, \cdots, 28.913, 17.022, \cdots ,17.022,10.466, \cdots ,10.466]^T$$

$$x = [x_1 , x_2 , \cdots , x_{19},\ x_{20}]^T$$

	MQ-1 Predator								MQ-9 Reaper								RQ-4 Global Hawk					
decision varables	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	Minimum Number of UAV	
coefficients	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	
solution	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0		
constraints	Matrix A																					vector b
1	0	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
2	0.087	0	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
3	0.087	0.087	0	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
4	0.087	0.087	0.087	0	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
5	0.087	0.087	0.087	0.087	0	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
6	0.087	0.087	0.087	0.087	0.087	0	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
7	0.087	0.087	0.087	0.087	0.087	0.087	0	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
8	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
9	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
10	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	0	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
11	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	0	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
12	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	0	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
13	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	0	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
14	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	0	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
15	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	0	11.978	36.554	36.554	36.554	36.554	73.109	17.022
16	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	0	36.554	36.554	36.554	36.554	73.109	17.022
17	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	0	36.554	36.554	36.554	36.554	10.446
18	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	0	36.554	36.554	36.554	10.446
19	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	0	36.554	73.109	10.446
20	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	0	73.109	10.446

Figure 21. Deterministic model : $K = 20$, $d = 1{,}100$ miles

The obtained linear program model can be easily solved using Microsoft Excel Solver. The optimal cyclic schedule consists of 2 RQ-4 Global Hawk's.

Next we assume that there are no RQ-4 Global Hawk UAVs because they have already been assigned to another mission and the resulting fleet consists only of 16 UAVs – 8 MQ-1 Predator's and 8 MQ-9 Reaper's. With the target being 1,100 miles away, the optimal cyclic schedule is found to consist of three MQ-9 Reaper's. The matrix A is

$$A = \begin{bmatrix} 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 \\ 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 \\ \vdots & \ddots & \vdots & & \ddots & \vdots \\ 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 \\ 0.087 & \cdots & 0.087 & 11.978 & \cdots & 11.978 \end{bmatrix}$$

The following figure obtained from MS Excel Solver summarizes the results.

	MQ-1 Predator								MQ-9 Reaper									
ecision varables	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	inimum Number of UAV	
coefficients	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	
solution	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0		
constraints	Matrix A																	vector b
1	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
2	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
3	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
4	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
5	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
6	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
7	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
8	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	28.913
9	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
10	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
11	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
12	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
13	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
14	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
15	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022
16	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	35.934	17.022

Figure 22. Deterministic model: $K = 16$, $d = 1{,}100$ miles

4.3 Stochastic Modeling Applications

This section illustrates an application of the risk neutral and sensitive models to a different UAV fleet. Assume that the operating base has 16 available UAVs with 14 MQ-1 Predator's and 2 MQ-9 Reaper's and that the target area is 1,000 miles away from the operating base. Next, assume that the MQ-1 roundtrip time is normally distributed with mean 26.8 hours and standard deviation 2.2 hours, and that the MQ-9 roundtrip time is also normally distributed but with mean 15.9 hours and standard deviation 1.9 hours. The constant loitering times are 2.3 and 13.1 hours respectively. The following table summarizes the data.

Table 5. UAV Data (hours) : $d = 1{,}000$ Miles

	Loitering Time (Constant)	*Roundtrip Time (Normal Dist.)*	
		Average	*Standard Deviation*
MQ-1 Predator	2.3	26.8	2.2
MQ-9 Reaper	13.1	15.9	1.9

First, let us apply the risk neutral model which closely resembles the deterministic one. The model, which has 16 decision variables, is

$$\min\ z = \sum_{i=1}^{16} x_i$$

subject to

$$\sum_{j=1, j\neq i}^{16} a_j x_j \geq \mu_i, \quad i = 1,2,\ldots,16$$

$$x_i = \{0, 1\}\,.$$

The following figure, which is a screen shot from Microsoft Excel Solver, shows the matrix A and the vectors b and x.

	MQ-1 Predator														MQ-9 Reaper			
decision varables	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	Minimum Number of UAVs	
coefficients	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	
solution	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1		
constraints	Matrix A																	vector b
1	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
2	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
3	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
4	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
5	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
6	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
7	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
8	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	26.739
9	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	13.071	13.071	28.403	26.739
10	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	13.071	13.071	28.403	26.739
11	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	13.071	13.071	30.664	26.739
12	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	13.071	13.071	30.664	26.739
13	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	13.071	13.071	30.664	26.739
14	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	13.071	13.071	30.664	26.739
15	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	13.071	17.593	15.929
16	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	0	17.593	15.929

Figure 23. Risk Neutral Model : K = 16, d = 1,000 miles

Using Microsoft Excel Solver we find that the optimal schedule has 4 UAVs − 2 MQ-1 Predator's and 2 MQ-9 Reaper's.

Next, we illustrate the risk sensitive (chance-constrained) model with a numerical example. Here, the loitering time is a constant as given in Table 5 and the threshold

probability α is set to 0.95. It follows that K_{α} is 1.645 and the linear program obtained from the chance-constrained program is

$$\min\ z = \sum_{i=1}^{16} x_i$$

subject to

$$\sum_{j=1, j\neq i}^{16} a_j x_j \geq \mu_i + 1.645\sigma_i, \quad i = 1,2,\ldots,16$$

$$x_i = \{0, 1\},$$

where the matrix A is the same as in the previous model because it depends only on the loitering but the vector b has changed as shown in the following screen shot obtained from MS Excel Solver.

	MQ-1 Predator														MQ-9 Reaper			
decision varables	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	Minimum Number of UAVs	
coefficients	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	5	
solution	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	1		
constraints	Matrix A																	vector b
1	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
2	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
3	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
4	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	30.664	30.358
5	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
6	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
7	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
8	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
9	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
10	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	2.261	13.071	13.071	32.925	30.358
11	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	2.261	13.071	13.071	30.664	30.358
12	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	2.261	13.071	13.071	30.664	30.358
13	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	2.261	13.071	13.071	32.925	30.358
14	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	13.071	13.071	32.925	30.358
15	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	0	13.071	19.854	19.054
16	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	2.261	13.071	0	19.854	19.054

Figure 24. Risk Sensitive Model : $d = 1{,}000$ miles, $\alpha = 0.95$

In this model the optimal solution consists of 5 UAVs − 3 MQ-1 Predator’s and 2 MQ-9 Reaper’s. Moreover, as α increases the size of the optimal schedule also increases. For example, if α goes up to 0.99 then the size of the optimal schedule goes up to 6 showing an increase of one MQ-1 Predator from the previous optimal solution. The

following chart shows how the probability α affects the number of UAVs in the optimal scheduling solutions.

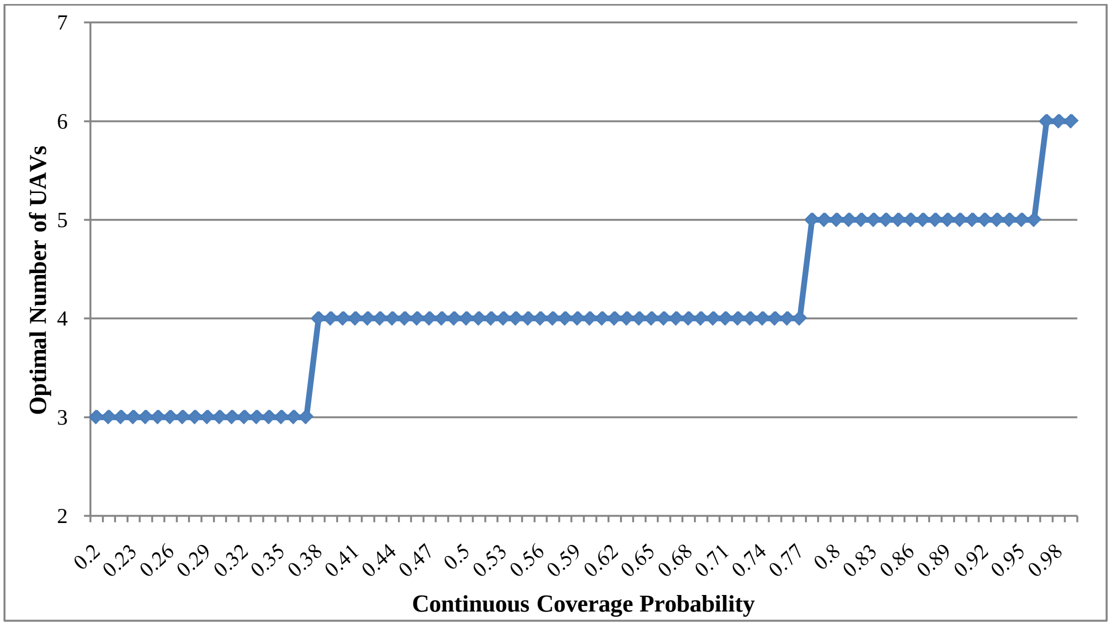


Figure 25. Optimal Number of UAVs vs. Threshold Coverage Probability

This result shows that as the threshold probability α increases the optimal number of UAVs also increases. This is expected because the optimal number of UAVs should go up as we require a higher probability of providing continuous coverage. In other words, as we reduce the risk of violating the continuous coverage constraints the required number of UAVs should go up. The technical justification for that is simple and follows from the fact that the feasible region defined by

$$\sum_{j=1, j\neq i}^{K} a_j x_j \geq \mu_i + K_{\alpha_i}\sigma_i, \qquad i \in \{1,2,\ldots,K\}$$

becomes smaller as α_i goes up.

Next we show how the minimum number of UAVs varies as a function of the distance d. Here for the risk sensitive model we let the threshold probability be $\alpha = 0.95$.

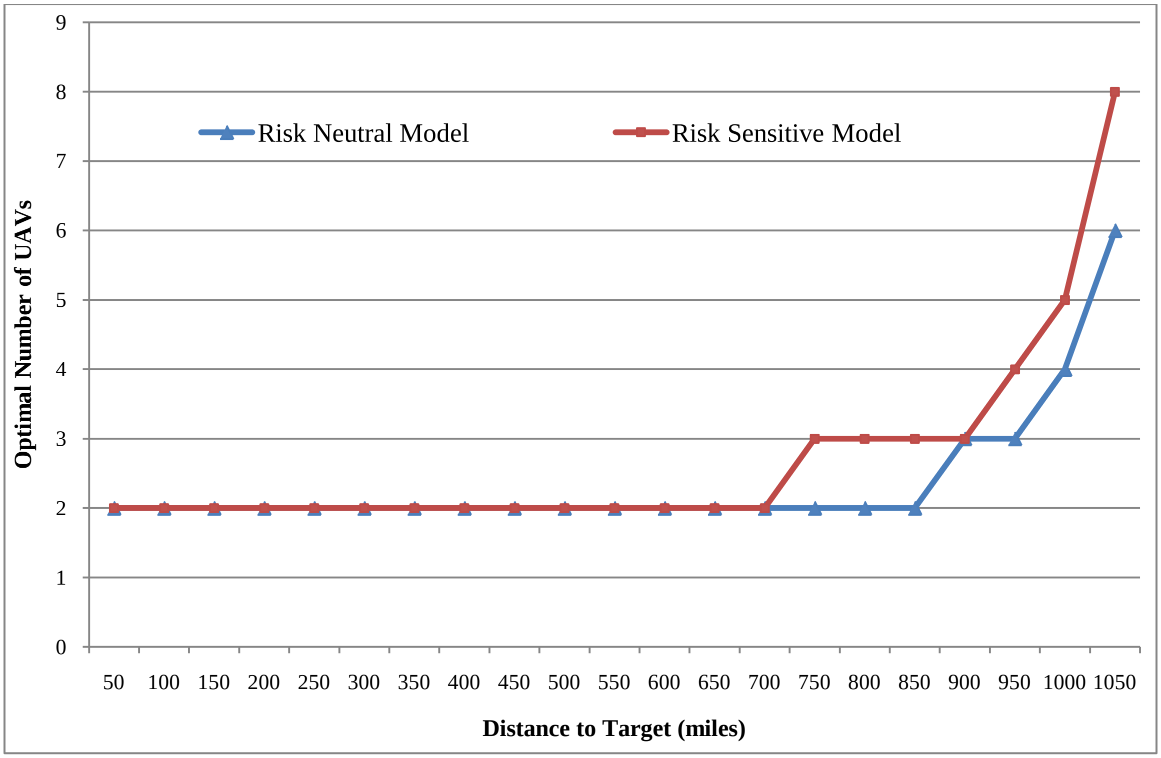


Figure 26. Risk Neutral & Sensitive Models

Figure 26 shows the optimal schedule size for both the risk sensitive and the risk neutral models as a function of the distance to the target area. Observe that the risk sensitive curve is on top of the risk neutral curve. That can be explained by the facts that

$$\sum_{j=1, j\neq i}^{K} a_j x_j \geq \mu_i + K_{\alpha_i}\sigma_i, \qquad i \in \{1,2,\ldots,K\}$$

implies the constraint

$$\sum_{j=1, j\neq i}^{K} a_j x_j \geq \mu_i, \qquad i \in \{1,2,\ldots,K\},$$

the feasible region of the risk neutral model contains that of the risk sensitive model, and both have the same minimizing objective function.

Note that as the distance to the target area decreases the roundtrip time of a UAV also decreases and this implies that for both models the number of UAVs in an optimal schedule goes down. We know that the smallest and best optimal schedule is of size two because continuous coverage cannot be obtained with one UAV. As a result, as the roundtrip time becomes smaller and smaller both models will eventually exhibit an optimal schedule of size two. This explains why the curves in Figure 26 coincide for a while at the 2 UAV level.

4.4 Conclusion

The numerical examples discussed in this chapter are simple illustrations of the basic mathematical programming formulation of the continuous coverage problem. Real applications that can benefit the warfighter need to wait until more realistic and practical features of the coverage problem are introduced and studied. The main purpose of this study was to lay the ground for further investigations that may lead to meaningful applications. We believe that both the deterministic and stochastic versions have a high potential to lead to fruitful applications. However, the stochastic version will very likely need an advanced background in stochastic programming.

Chapter 5

V. CONCLUSION

5.1 Introduction

This research provides an original approach to the continuous coverage problem by developing a new mathematical model to serve as a baseline model for further UAV scheduling studies. As a result, various extensions of the model are possible but we only mention a couple of them.

5.2 Future Research Directions

As the UAV fleet size increases the complexity of the problem increases and because of the special form of the objective function, there are generally a large number of optimal cyclic schedules of the same size. Therefore it would worthwhile to introduce a new metric that can differentiate these optimal schedules obtained earlier. One such metric is the slack time of a UAV within a cyclic schedule. Based on the definition of a slack time it is desirable that a UAV has a large slack time within a schedule to give it time to remedy any contingency that may arise before deployment to the target area. Therefore a cyclic schedule with large UAV slack times is more efficient. A metric that could measure the value of a schedule is its total UAV slack time. This metric can be used to formulate a new objective function for the zero-one integer program. Weight or priority coefficients may also be introduced to refine the objective function.

Another possible extension is to use an aerial tanker to refuel the UAVs. The tanker will

be closer to the target area than to the operating base for this option to make sense. The UAVs will fly for a shorter time to refuel and their productivity will increase by using the extra time gained to provide more loitering time. It follows that a UAV will be commuting between the target area and the tanker but once in a while the UAV needs to return to the operating base for maintenance to keep its performance up. In this case, a UAV will have 2 roundtrip variables involving the tanker and the operating base. In this approach the safety of the tanker becomes a major concern. The distance between the tanker and the target area is critical and needs to be evaluated carefully. The tanker may also be supporting several missions and the location of the tanker is another variable needing careful analysis. Taking into account the risks and rewards the critical location of the tanker can be determined. The rewards being that a smaller number of UAVs will be needed. A risk analysis needs to be done.

5.3 Conclusion

This thesis has introduced a new class of cyclic scheduling problems with a prototype being the UAV continuous coverage problem. A mathematical framework was initiated to serve as a stepping stone to further the study of this class of problems. Various theoretical as well as practical results were derived and used to formulate the basic problem as a mathematical programming problem. The next step is to build on and extend this framework by adding more features to handle more complex problems of the same kind.

Appendix A. Blue Dart

UAV Continuous Coverage Pays Off!

USA today, June 15, 2009 wrote “Lt. *Gen. David Deptula, Air Force Deputy chief of staff for intelligence, surveillances and reconnaissance missions, said intelligence gathering is key to counterinsurgency operations. An example, he said in an interview, was the tracking and killing in 2006 of Abu Musab al-Zarqawi, the leader of al-Qaeda in Iraq. It took 600 hours of surveillance by a Predator drone to track Zarqawi and a matter of minutes for an F-16 to drop the bombs that killed him*”. This excerpt stresses the importance of UAVs in supporting the warfighter… A redoubtable enemy has been eliminated. How did the UAVs do it? How did they provide continuous coverage of the target? How many UAVs were there? One, two, three …? How do you find the right number? How were they scheduled?

The UAVs performed 600 hours of continuous coverage to eliminate a redoubtable enemy! Continuous coverage was the key to the operation success! Recently we developed a mathematical model to optimize the process of choosing the best UAV cyclic schedule to provide continuous coverage of a target area.

The problem to be solved can be easily understood. A critical ISR mission requiring continuous surveillance and coverage of a target area is to be accomplished using UAVs as the main resource. A UAV fleet is available at the operating base to support the mission. The UAV being a valuable and scare resource is to be frugally used particularly when there are other ISR missions around the world requiring UAVs. The main questions to answer are how to sequence the UAVs to conduct the mission to

provide continuous coverage of the target area and how many UAVs are required knowing that one should not use more UAVs than needed. Continuous coverage is a key requirement. Ideally, if possible, there should not be coverage gaps at all since that may render the mission worthless; for example, objects of interest may move out of the target area without them being detected. There could be several unforeseen events that could prevent continuous coverage and if it is the case then one needs to obtain the maximum coverage possible.

A new mathematical framework was needed to solve the problem. We developed the framework from scratch since no previous work was done on such a problem. It will serve as a baseline model for more complex UAV scheduling problems. We introduced the notion of a UAV cyclic schedule and in the case of a homogeneous UAV fleet we derive a formula for the minimum of UAVs needed to ensure continuous coverage. For a non-homogeneous UAV fleet we formulate the problem as a binary integer program and solved it. We built an Excel tool based on the findings. Taking a fleet of MQ-1's, MQ-9's and Global Hawk's we used the tool to come up with the optimal UAV cyclic schedule that provides continuous coverage.

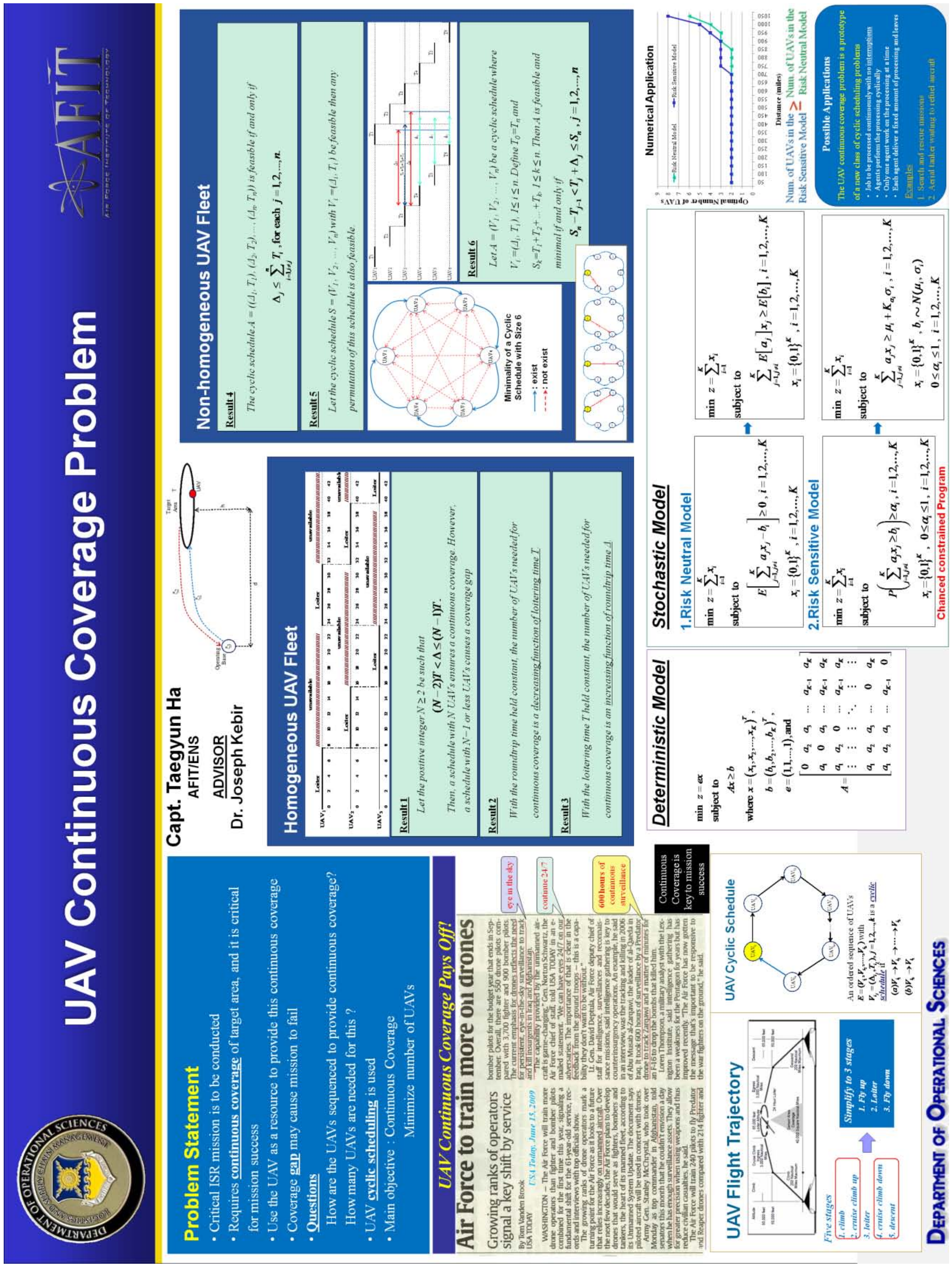
Several key insights were obtained. The model provides valuable information on the parameters driving the UAV performance coverage. Loitering and transit times are the most impacting parameters driving the performance coverage of the UAVs and the needed number of UAVs goes up as the transit time goes up. Also, the number of needed UAVs goes down as the loitering time goes up. A new UAV productivity metric is introduced as the ratio of the loitering to the transit time. As this ratio goes up a smaller

number of UAVs are required to provide continuous coverage because they are more productive. The results obtained can be applied to other surveillance problems and particularly those pertinent to NRO and NSA.

The developed mathematical model can be used to solve other problems sharing the same structure as the UAV continuous coverage problem. The model that we developed can be applied to other situations where a task is to be processed continuously without interruptions and the "agents" providing the resources to perform the task are scheduled cyclically. Each agent carries out a portion of the task before handing it over to the next one. Here the agent is limited in its capability to work for a long time without interruption because it needs resources to sustain itself while working and so needs to break away from the task while another agent takes over. Only one agent can work on the task at a time. Search and rescue missions where continuous coverage may be crucial to find survivors, aerial tankers needing to orbit while waiting to refuel aircraft, satellite orbiting to provide a continuous flow of information may be modeled using the obtained results.

UAV continuous coverage of a target is crucial for the success of a critical ISR mission. Finding the minimum number of UAVs required and an optimal cyclic schedule to ensure continuous coverage will enhance the asymmetric warfare capabilities of the Air Force.

Appendix B. Poster Chart



Appendix C. Excel Model User Guide

The software implementation is in the form of a Microsoftr Office Excelr spread-sheet Microsoftr Corporation (2006). First we calculate the loitering and roundtrip times using the endurance-time and distance to the target data and then we build the linear programming model to be solved using Excelr Solver.

Step 1. Data Setting

The loitering and roundtrip times are calculated using the UAV endurance time, cruise speed, and distance data.

	endurance	speed				
MQ-1	24	92	mile/h	Refueling and Maintenance time	5 hours	
MQ-9	24	183	mile/h	Distance to Target Area	1100 mile	1770.3 km
RQ-4	42	404	mile/h			

	$\xi 1$	$\xi 2$	$\xi 3$	Δ	T
MQ-1	11.957	11.957	5	28.913	0.087
MQ-9	6.011	6.011	5	17.022	11.978
RQ-4	2.723	2.723	5	10.446	36.554

Figure 27. Basic Data Setting Spread Sheet

Step 2. Linear Programming Model Setting

Using the data calculated from step 1 we build the linear program model to solve using Excelr Solver.

= Sumproduct(B3:U3, B4:U4)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1		MQ-1 Predator								MQ-9 Reaper								RQ-4 Global Hawk					
2	decision varables	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15	x16	x17	x18	x19	x20	Minimum Number of UAVs	
3	coefficients	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	
4	solution	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0		
5	constraints	Matrix A																					vector b
6	1	0	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
7	2	0.087	0	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
8	3	0.087	0.087	0	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
9	4	0.087	0.087	0.087	0	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
10	5	0.087	0.087	0.087	0.087	0	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
11	6	0.087	0.087	0.087	0.087	0.087	0	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
12	7	0.087	0.087	0.087	0.087	0.087	0.087	0	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
13	8	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	28.913
14	9	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
15	10	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	0	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
16	11	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	0	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
17	12	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	0	11.978	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
18	13	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	0	11.978	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
19	14	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	0	11.978	11.978	36.554	36.554	36.554	36.554	73.109	17.022
20	15	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	0	11.978	36.554	36.554	36.554	36.554	73.109	17.022
21	16	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	0	36.554	36.554	36.554	36.554	73.109	17.022
22	17	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	0	36.554	36.554	36.554	36.554	10.446
23	18	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	0	36.554	36.554	36.554	10.446
24	19	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	0	36.554	73.109	10.446
25	20	0.087	0.087	0.087	0.087	0.087	0.087	0.087	0.087	11.978	11.978	11.978	11.978	11.978	11.978	11.978	11.978	36.554	36.554	36.554	0	73.109	10.446

= Sumproduct($B$3:$U$3, B25:U25)

Figure 28. Screen Shot of Excel Spread-sheet

<u>*Decision variables*</u> : A binary variable indicates whether or not a UAV is a part of cyclic schedule.

<u>*Coefficients*</u> : Each decision variable has a coefficient 1 in the objective function.

<u>*Solution*</u> : Optimal solution vector $x^* = (x_1^*, x_2^*, \cdots x_{20}^*)$

<u>*Matrix A*</u> : Entries of A are all UAV loitering times with the main diagonal entries being zero.

<u>*Vector b*</u> : Each entry of b is a UAV roundtrip.

Step 3. Add-in Excelr Solver

1. Click the Microsoft Office Button, and then click Excel Options.
2. Click Add-Ins, and then in the Manage box, select Excel Add-ins.

3. Click Go.

4. In the Add-Ins available box, select the Solver Add-in check box, and then click OK.

5. After you load the Solver Add-in, the Solver command is available in the Analysis group on the Data tab.

Step 3. Formulating Linear Program

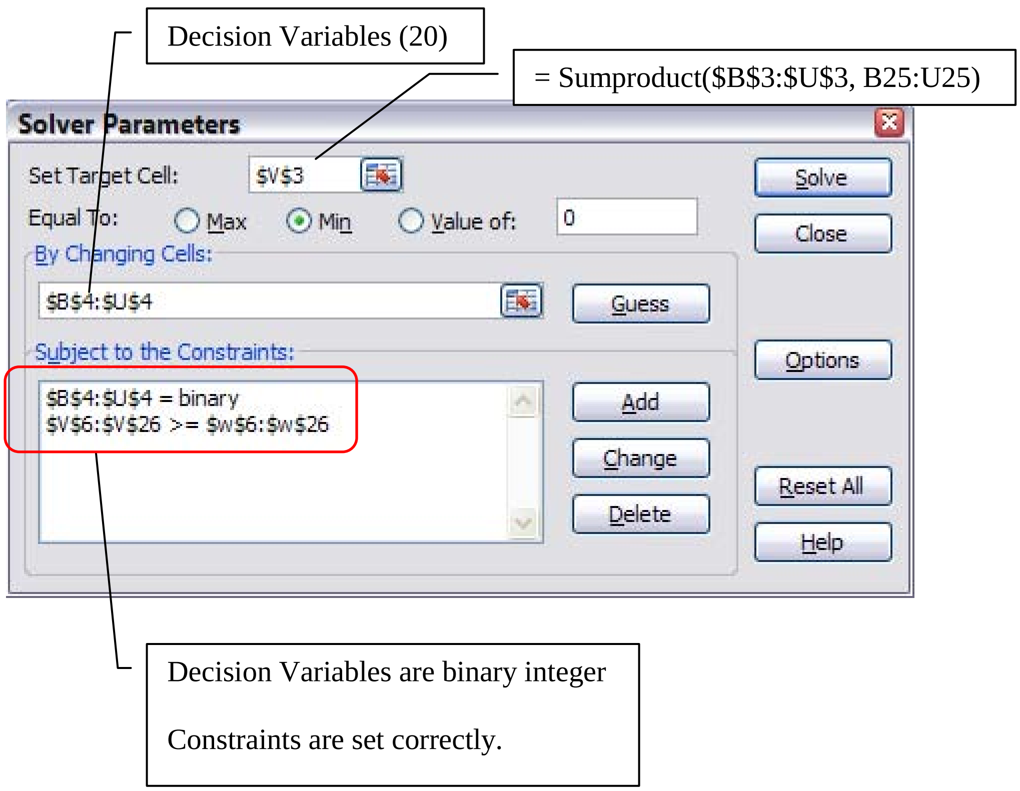


Figure 29. Solver Parameters

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14. ABSTRACT : The purpose of this research is to develop a method to find an optimal UAV cyclic schedule to provide maximum coverage over a target area to support an ISR mission. The goal is to reach continuous coverage. UAV continuous coverage of a target area is crucial for the success of an ISR mission. Even the smallest coverage gap may jeopardize the success of the mission. Ideally it is desirable to obtain continuous coverage of a target area but the stochastic nature of the problem makes continuous coverage without gaps unlikely. However, it is still possible to obtain a high coverage rate. Coverage gaps may occur at handoff from one UAV to another. We first study a deterministic model with identical UAVs and derive the minimum number of required UAVs to ensure continuous coverage. Continuous coverage is possible only in the deterministic setting. The model provides valuable insights on the parameters driving the UAV performance coverage. It is shown that the loitering and the roundtrip times are the most impacting parameters driving the performance coverage of the UAVs. It is proved that the number of UAVs is an increasing function of the roundtrip time and a decreasing function of the loitering time. The results obtained for the model with identical UAVs are then extended to the deterministic model with possibly non-identical UAVs. Conditions for continuous coverage are derived and used to formulate the continuous coverage problem as an integer linear program. When the UAV data is stochastic the problem is formulated as a chance constrained program and converted under suitable conditions to a deterministic integer linear program. Some numerical applications and extensions of the models are discussed.

15. SUBJECT TERMS
UAV, Cyclic scheduling, Continuous coverage, Loitering, Roundtrip, Deterministic Model, Stochastic Model, Linear Program, Stochastic Program, Chance-Constrained Program

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